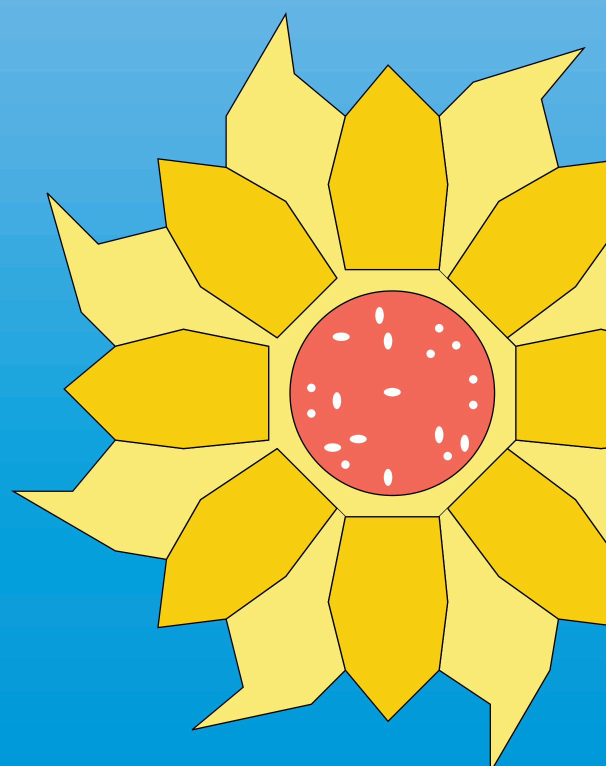
Geometry Activities

for Middle School Students

with THE GEOMETER'S SKETCHPAD®



Karen Windham Wyatt

Ann Lawrence

Gina M. Foletta

Key Curriculum Press

BLACKLINE MASTERS FOR USE WITH THE GEOMETER'S SKETCHPAD

Geometry Activities

for Middle School Students

with THE GEOMETER'S SKETCHPAD®

WINDOWS | MACINTOSH

Karen Windham Wyatt
Ann Lawrence
Gina M. Foletta

KEY CURRICULUM PRESS
Innovators in Mathematics Education

BLACKLINE MASTERS FOR USE WITH THE GEOMETER'S SKETCHPAD

Editors Dan Bennett and John Bergez
Editorial Assistant Romy Snyder
Advisor William Finzer
Reviewers Ellen M. Cooch, David Rock, Mary Ann Rota, Susan Smolin, Anne C. Thompson
Production Editor Jason Luz
Production Consultant Steve Rogers
Production Coordinator Diana Krevsky
Copyeditor Mary Roybal
Cover and Interior Design Kirk Mills
Composition and Art Kirk Mills

Publisher Steven Rasmussen
Editorial Director John Bergez

Photo credit: Photo on page ix supplied by authors. Pictured (left to right) are Ann Lawrence, Charlie Adams, Brittany Beth, Karen Wyatt, and David Sloan.

Key Curriculum Press
1150 65th Street
Emeryville, CA 94608
510-595-7000
http://www.keypress.com

Distribution in Ontario by:
Spectrum Educational Supplies
125 Mary Street
Aurora, Ontario L4G 1G3
1-800-668-0600
speedu@ibm.net
http://www.spectrumed.com

About the Authors

At various times during the last twenty years, Karen Windham Wyatt has taught middle school mathematics, high school mathematics, and computer programming. For the last four years, she has served as Secondary Mathematics Technology Facilitator for Mountain Brook City Schools. She has been a co-instructor of three five-day Beginning Sketchpad institutes, taught numerous calculator and computer workshops, and served as co-editor for the software review feature of the journal *Mathematics Teaching in the Middle School.* With Ann Lawrence, she has co-authored and received several grants along with local, state, and national recognition. Her daughter hopes that someday she can last twenty-four hours without touching a calculator or computer! She can be contacted via email at WYATTK@MTNBROOK.K12.AL.US.

Ann Lawrence has taught middle school mathematics for over twenty years. During that time, she has undertaken a wide range of related endeavors, from co-coaching a national championship MATHCOUNTS team to being Cartoon Editor for the journal Mathematics Teaching in the Middle School, a T^3 instructor for graphing calculators, and a member of the K-8 Leadership Institute for Discrete Mathematics. For the past five years she has been coordinator of the Mountain Brook Junior High mathematics department. With Karen Wyatt, she has co-authored and received several grants along with local, state, and national recognition. For fun, she loves to travel and go white water-rafting, and she aspires to be a serious photographer someday (in her spare time?)! She can be contacted via email at AnnLTeach@AOL.com.

Gina M. Foletta, Ph.D., has taught high school mathematics in California and Iowa. She is currently Assistant Professor of mathematics at Northern Kentucky University. Gina has worked extensively with pre-service and in-service teachers, especially in the areas of geometry and technology. She is currently co-directing the Kentucky Partnership for Reform Initiatives in Science and Mathematics (PRISM) Secondary Mathematics Initiative in northern Kentucky. Underlying all of her work is an interest in appropriate uses of technology in the learning and teaching of mathematics coupled with mathematical reasoning and thought. She can be contacted via email at foletta@nku.edu.

Contents

Unit 4: Quadrilaterals

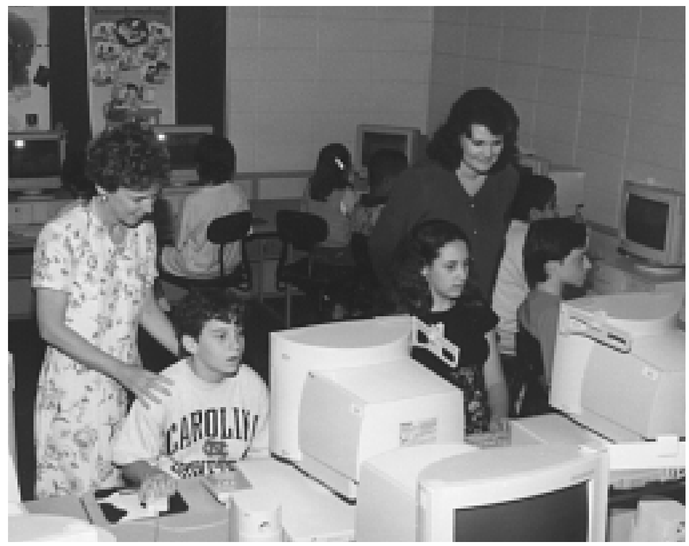
Unit 5: Symmetry

Unit 6: Transformations

Unit 7: Constructions

A Note to Teachers

Interactive computer software provides abundant opportunities to enhance the way we teach mathematics. Beyond the obvious advantage of electronic practice, software programs increase the options for presentation of new material and independent student investigations. Perhaps the possibilities have expanded most for the teaching of geometry. Using The Geometer's Sketchpad®, for example, students can explore concepts much more easily than in the past. The ability to create and manipulate figures on the computer screen enables students to quickly visualize and produce many examples, examine properties of the figures, look for patterns, and make conjectures. This capability is especially exciting because these steps are essential for most students to truly understand even the basic concepts of geometry.



As teachers of middle school students, we deal with students at the age where they are beginning to think abstractly; also, the mathematics curriculum for this age group includes the introduction of many of the concepts that lay the foundation for the formal study of geometry. Thus, it is during these years that students need exactly the kind of experiences that The Geometer's Sketchpad makes possible.

We believe that students should discover for themselves as many of the ideas and relationships of geometry as possible. Of course, when first encountering this approach students need more guidance down the path to discovery than they do later. Nevertheless, we have found that carefully constructed activities enable our middle school students to experience the joy of discovering mathematics, the confidence that comes with success, and the framework for a pattern of independent learning.

With these experiences in mind, we have developed the units in this book for teachers to use with The Geometer's Sketchpad in middle school mathematics classes. The materials are designed to be flexible. They may be used as the basis for an independent geometry unit for a middle school, as a set of activities for reinforcement or enrichment for a text-based unit, or as individual activities to be woven into the mathematics curriculum on appropriate occasions.

Content and Organization

Units 2 through 6 each focus on a cluster of geometric concepts that are usually introduced during the middle school years. They are grouped as follows:

Unit 2: Points, Lines, and Angles
Unit 3: Triangles
Unit 4: Quadrilaterals
Unit 5: Symmetry
Unit 6: Transformations

For each of these units, the material is self-contained. Of course, there are activities in each unit that integrate concepts from others, but the primary emphasis is indicated by the unit title. None of the units rely on students having done any other unit, although the Teacher Notes sometimes refer to related activities that might be helpful to do first.

Unit 1 contains tours for getting acquainted withThe Geometer's Sketchpad® and some of its animation capabilities. Unit 7 contains step-by-step instructions for a variety of constructions that use Sketchpad and are suitable for middle school students.

Units 2 through 6 include the following components.

1. **Teacher Notes.** The Teacher Notes in each unit include an anecdotal commentary with hints or comments about each activity and project, a list of mathematical concepts, a list of prerequisite Sketchpad skills, definitions of essential vocabulary, and detailed instructions for carrying out Teacher Demonstrations. Also included are writing prompts for student assignments, examples of student work, and answers for both the guided discovery activities and Wrap-Up assignment.

   Sketches for Teacher Demonstrations are included in a folder on the disks that accompany this book. You may often find the demonstrations helpful in orienting students before they begin some of the independent activities. The demonstrations help you consider prerequisites, present appropriate mathematical terms, and introduce or review Sketchpad skills that will be employed in the activities.

2. **Activities and Projects.** The activities in each unit provide a framework for students' explorations. Often there are one or two activities requiring very little technical expertise that you will feel comfortable giving your students with little or no guidance beforehand. Other activities maximize guided discovery by students. Some of these are more open-ended than others, and their difficulty levels vary. You should feel free to choose those activities that are most appropriate for the ability and skill levels of your students. In some instances, even though the activities are designed for use with a class of students working alone or in pairs, using the activity as a teacher demonstration will be appropriate. Sketches for these activities are included on the accompanying CD-ROM, while the text provides blackline masters for student work.

   Each unit also contains projects, which are designed to help students explore the unit topic further in small groups or independently. Usually the projects require students to create their own sketches. Each project specifies an objective to accomplish or an area to explore, but specific instructions are not given.

3. **Wrap-Up.** The unit Wrap-Up is a blackline master you can use for individual, in-class reinforcement, as a homework assignment, or as an assessment instrument. The Wrap-Up does not require the use of Sketchpad.

4. **Quick Reference Guide.** These step-by-step instruction sheets enumerate Sketchpad maneuvers needed to carry out the activities in each unit. We have found it helpful to copy a classroom set of each of these reference guides and laminate them. We leave the laminated copies in the computer lab so that students can use them not only when working with their classmates but also when they go to lab for independent explorations with Sketchpad.

Hints and Suggestions

Especially if you're new to Sketchpad, we suggest working through the Tours, Quick Reference Guides, and Teacher Demonstrations in your preparation for working with students. Becoming familiar with these aids can greatly facilitate your own learning of the software and increase your comfort level with these materials in the classroom or computer lab.

But even if you're using Sketchpad for the first time, don't let your inexperience keep you from placing the software into the hands of your students! One of the best aspects of our first year in the middle school classroom with Sketchpad was learning along with our students. Students and teachers learned and taught one another not only the software but also mathematics. Nothing in the classroom can compare to the excitement generated when students and teacher both experience the joy of learning mathematics together.

If you are an adventurous teacher, you may choose to alter or omit the Teacher Demonstrations altogether. Another choice is to let students explore for themselves with the Demo sketches. In our first year we certainly had no notes or predesigned sketches, yet the year was exciting as well as productive.

Regardless of the degree of structure you are comfortable with, be sure to allow time for students to simply explore with the software. Before or after initial activities students love to try the tools and experiment on their own. This free exploration motivates students to want to learn more about the capabilities of Sketchpad, thus drawing forth a rich learning atmosphere.

Finally, we encourage you to allow your students to work in pairs or in small groups for some if not most of the activities. The units in this book, as well as Sketchpad itself, are ideal instruments for cooperative learning. When we teach this way, our students exhibit both increased mathematical understanding and greater Sketchpad proficiency.

Some Comments on Sketchpad Sketches and Techniques

In constructing the sketches that accompany the demonstrations and activities, we have sometimes introduced constraints for pedagogical reasons. We don't want students hampered by their lack of mathematical understanding or Sketchpad proficiency. For example, when the goal is for students to discover the properties of the various types of quadrilaterals, they obviously don't have enough knowledge to construct a rectangle that has the mathematical properties of a rectangle, or they wouldn't need to do the activity! By dragging a rectangle that is constructed to remain a rectangle, they can test to see whether it can be dragged into a square with the confidence that their results are mathematically sound.

Also, some of the sketches have orientation restrictions so that students can avoid distractions and focus on the target concept. In either case, as your students become more experienced with Sketchpad, you may want them to devise more of their own constructions. At some point, you may want to redesign some of our sketches to take advantage more fully of the dynamic features of the software.

There are many quick keys and other shortcuts built into Sketchpad, and these can be terrific time-saving measures. Hints on shortcuts are provided in many of the Quick Reference Guides. However, in our experience with middle school students it is better to postpone the introduction of most shortcuts, including quick keys, until the students are somewhat familiar with the software.

The primary benefits of waiting are threefold. First, middle school students who are new to Sketchpad often need some experience before they can carry out shortcuts successfully. What seems to be a simple maneuver at a later time can be quite difficult in the beginning. For example, to construct a segment between two points, a Sketchpad user can use the Segment tool, click on one point, and drag and release on the other point instead of selecting the two points, going to the Construct menu, and choosing Segment. However, when many middle schoolers try to perform this shortcut, they fail to drag the segment near enough to the second endpoint to activate the "snap to" feature. As a result, their figure falls apart eventually, and they are both disappointed and frustrated. Of course, with some experience using Sketchpad, these obstacles disappear.

Second, the speed of the shortcuts, especially during teacher demonstrations, may not provide students with enough time to process the steps the shortcut replaces. For example, use of the quick key for constructing the midpoint of a segment may be misconstrued as constructing an arbitrary point on the segment. We have found it works far better for students to discover this shortcut on their own or to show them the shortcut later after they clearly understand what constructing a midpoint means mathematically and can then realize what steps the shortcut replaces.

Third, the visual reinforcement of seeing the labels in the menus for the Sketchpad maneuver reinforces the students' grasp of the mathematical processes they are performing. For example, when designating a segment as a mirror, the user selects the segment, goes to the Transform menu, and chooses Mark Mirror. Going through this use of the menu seems to remind students why the segment is being selected and to help them connect the use of a mirror to the whole concept of transformations.

Sketches on the CD-ROM

As noted above, the CD-ROM that accompanies this book contains Geometer's Sketchpad sketches for the activities and Teacher Demonstrations. The CD is a hybrid disc that will work on computers running Microsoft Windows® and on computers running the Macintosh® operating system. The disc does not contain The Geometer's Sketchpad program itself. Sketchpad is available separately from Key Curriculum Press.

The sketches are organized in folders by chapter, with separate folders for activity sketches and demo sketches (except for the Quadrilaterals and Tours chapters, which each have only one folder of sketches). The Symmetry Demos folder also includes examples of student work on the symmetry project. Be sure to look these over—they're lots of fun.

Origins of This Book

This book originated in the experiences of two of the authors, Karen Wyatt and Ann Lawrence. Our Junior High Math Department was working to implement the NCTM Curriculum and Evaluation Standards. We were making progress, but much more slowly than we wished. We took a proposal to our principal and superintendent in which we asked for release time for Karen to lead a push to integrate technology into the mathematics curriculum of all our courses. Once we received the go-ahead, all the teachers in our department helped collect ideas for calculator activities, computer topics, and software.

One of our teachers saw a demonstration of The Geometer's Sketchpad at a regional NCTM meeting in the spring of 1993. At the national convention in Seattle that same spring, Karen attended the first user group meeting for Sketchpad, and we were sold on the software! We wrote grant proposals and were granted a site license for Sketchpad.

Karen attended the first five-day Sketchpad Institute in the summer of 1993. The school administration granted Karen the partial release time we had requested and our year began. We used Sketchpad at all three grade levels with great success. Karen kept a journal during that year, so when Key requested feedback from the participants of the Sketchpad Institute, she sent them entries from the journal and copies of the instructions and activities that involved Sketchpad. Key was excited about the ways we were applying Sketchpad in the middle school classroom. They called to ask us whether we were interested in writing a book of middle school activities for Sketchpad. We were and we did!

We held a Sketchpad camp the first summer to refine some of the activities we had tried with our students the previous year and to try out new ideas. During the next two school years we continued the process of developing and refining activities as ideas occurred to us, our fellow teachers, and our students. Gina Foletta joined us for the final major round of revisions and the materials presented in this book were completed.

Without the help, support, and patience of the other teachers in our department, this book would never have been completed. Likewise, our students gave us inspiration to create and complete the task. Our team from Key—Bill Finzer, Dan Bennett, Steve Rasmussen, John Bergez, and Jason Luz—were our technical advisors and provided much encouragement. Kirk Mills supplied the interior and cover designs. We especially want to thank our daughters, Emily and Lori, who put up with ridiculous hours, endured computers even on vacations, and provided solicited and unsolicited editing advice. We are grateful to you all.

When all is said and done, this book is written for children, for that is where our dedication lies.

Karen Windham Wyatt
Ann Lawrence
Gina M. Foletta

Unit 1

TOURS

Teacher Notes

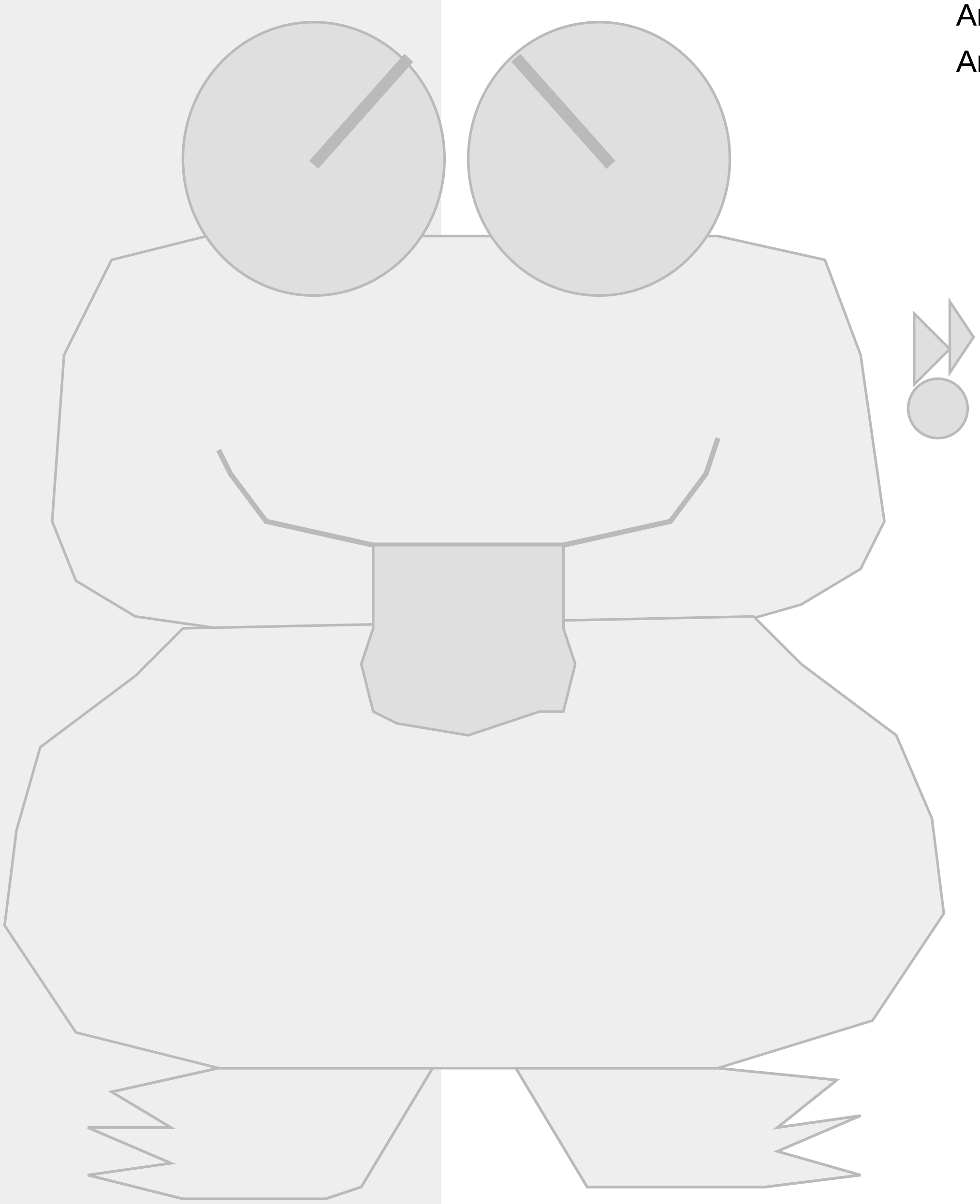
Getting to Know Sketchpad—Tour 1

Getting to Know Sketchpad—Tour 2

Animation—Tour 1

Animation—Tour 2

Animation—Tour 3



Commentary

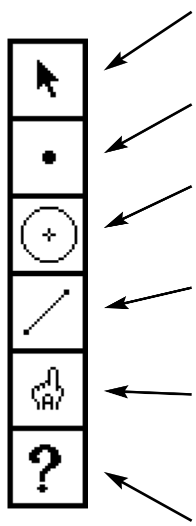
The purpose of the Tours unit is to provide step-by-step instructions so that students will feel comfortable using Sketchpad. **Getting to Know Sketchpad—Tours 1** and **2** will give middle-school students the hands-on experience they need as a foundation for building proficiency and confidence in using the software. We highly recommend having students work through these tours before they do other activities in this book.

Animation—Tours 1, 2, and **3** not only provide exposure to Sketchpad animation skills used in later activities, but also act as a springboard for independent student exploration. We have found the animation capabilities of Sketchpad exciting and extremely motivating for every group of middle-school students (and teachers!) with whom we have worked. Unlike the two introductory tours, the animation tours can be delayed until you feel they are appropriate. One excellent place to incorporate animation is with the project **Make Me Symmetric** in the Symmetry unit. Whenever you choose to use these tours, you may want to show your students some of our students' animated sketches (found in the Symmetry folder). Seeing other students' work has always sparked imaginations!

Getting to Know Sketchpad—Tour 1

Welcome to The Geometer's Sketchpad, an exciting and useful tool for students of all ages. With Sketchpad, you can explore geometry in ways you never would be able to otherwise. In **Getting to Know Sketchpad—Tour I,** you will learn some Sketchpad skills that you will use in doing activities and projects. Feel free to explore on your own with the tools as you proceed through the tour.

1. Open a new sketch by choosing **New Sketch** from the **File** menu**.** You will use this tour to explore some features of Sketchpad.
2. On the left side of your sketch, you'll see Sketchpad's toolbox, shown below. You will use most of these tools during this tour.



The Selection Arrow tool is used to select or deselect objects and to move objects.

The Point tool is used to construct points in your sketch.

The Circle tool is used to construct circles in your sketch.

The Segment tool is used to construct segments in your sketch. It can be changed to a Ray tool or a Line tool.

The Text tool is used to show or hide labels and to add text to your sketch.

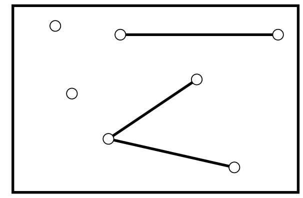
The Object Information tool ? displays a pop-up menu listing the selected objects. You can also click on an object with this tool to display information about the object.

The list of words across the top of your screen (see below) is known as the Menu Bar. You will use the various menus to choose commands.

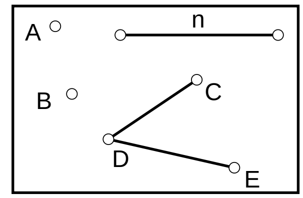
File Edit Display Construct Transform Measure Graph Work

3. Click on the Point tool and move onto your sketch. Click in a couple of places to construct points.
4. Click on the Segment tool and move onto your sketch. Press and hold down the mouse, and drag to draw a segment.

5. Now construct an angle using the Segment tool . First, draw a segment. Then, starting from one of the segment's endpoints, draw another segment to create an angle. Your screen should look similar to the figure at right.



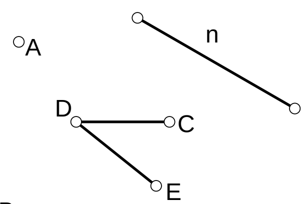
6. Click on the Text tool . To show a point's label, position the Text tool over the point and click. Show some of the point labels. Then click on the segment itself (not the endpoints) to show its label. Your screen should now look similar to the figure at right. Your labels may not be the same as the ones in this example.



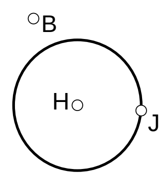
7. To move the objects or to change their size, click on the Selection Arrow tool . Using this tool, drag one of the endpoints of your segment to make the segment longer or shorter.

8. Drag one of the endpoints of a side of your angle to change the size of the angle.

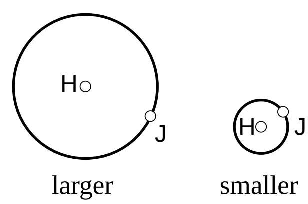
9. To move the segment, click on the segment itself. The segment should look similar to this when it is selected: . While it is selected, drag it. It should move around on your screen without changing its length.



10. Click on the Circle tool . Drag in your sketch to construct a circle. Notice that the center and a point on the circle are always given.

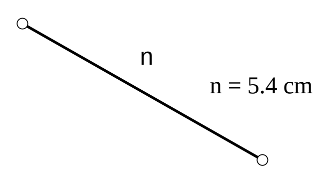


11. Click on the Selection Arrow tool and drag the point on the circle to change the circle's size. Also try dragging the center point to change the size of the circle.

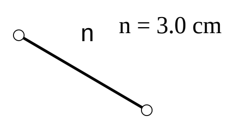


12. Drag the circle to change its location (drag the circle, not the point on the circle).

13. Click on the segment to select it again. Go to the **Measure** menu and choose **Length.** The measure for your segment should appear on your screen.

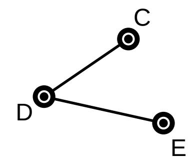


14. Drag one of the endpoints of your segment. The measure on your screen should change as you drag.

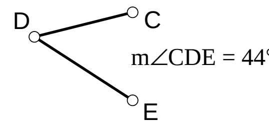


15. Choose the Selection Arrow tool, then click in any blank space on your screen. This click deselects everything. You need everything deselected so that you can select the necessary points to measure your angle.

16. Hold down the Shift key and click on the three points that define the angle, with the vertex point your middle selection. When you have them all selected, they should look similar to the diagram at right.

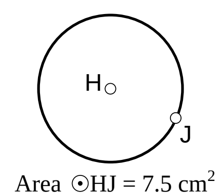


17. While the points are selected, go to the **Measure** menu and choose **Angle.** You should see the measure of your angle appear on your screen.



18. Click in any blank space on your screen to deselect all objects. Drag the endpoint of one of the angle's sides and observe the changing angle measure. Drag the angle into an acute angle, a right angle, and then an obtuse angle.

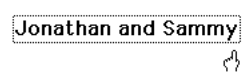
19. Click in any blank space to deselect objects. Click on the circle (not the point on the circle) to select it, then go to the **Measure** menu and choose **Radius, Circumference,** or **Area.** The measure you choose should appear on your screen.



Note: If any measure appears on your screen in an undesirable location, you may drag it to a different position on the screen.

20. Click on the Text tool and drag in a blank area at the top of your sketch to create a text box.

21. Type your name in this box.

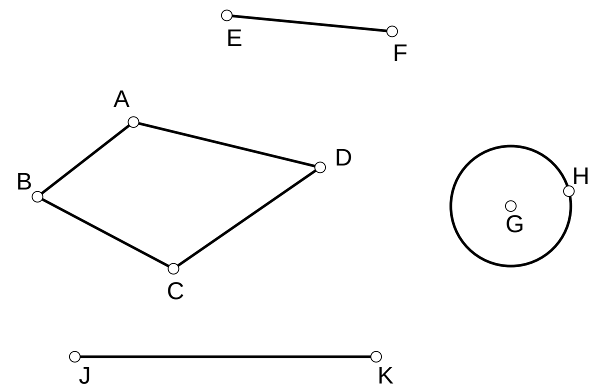


Summary

In this tour, you have learned to

- Use the Selection Arrow tool to select and move objects
- Use the Selection Arrow tool to deselect objects
- Use the Point tool to construct points in your sketch
- Use the Circle tool to construct circles in your sketch
- Use the Segment tool to construct segments in your sketch
- Use the Text tool to show labels and to add text to your sketch
- Change the size of objects in your sketch
- Measure objects in your sketch

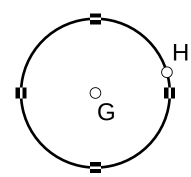
Getting to Know Sketchpad—Tour 2



1. Open the sketch Tour 2 (Mac) or Tour2.gsp (Windows). You will use this sketch to explore some other features of Sketchpad.

Use the Selection Arrow tool while working in steps 2–5.

2. To construct a segment between two given points, hold down the Shift key and click on points **E** and **F** in your sketch. Release the Shift key. Go to the **Construct** menu and choose **Segment.**
3. Click in any blank space in your sketch to deselect objects.
4. To construct a point on a circle, click on the circle in your sketch (not on point **H**). The circle should look like the figure at right when selected. Go to the **Construct** menu and choose **Point On Object.** A new point should appear on your circle. Drag the new point. Notice that it moves freely around the circle, but you cannot drag it off the circle. Drag point **H**. Because point **H** is one of the control points of the circle, dragging it will change the size of the circle.



5. Click on segment **JK,** go to the **Construct** menu, and choose **Point On Object.** Try dragging the new point that appears on $\overline{\mathbf{JK}}$. Notice that it will slide along $\overline{\mathbf{JK}}$ but you cannot move it off the segment.

Use the Text tool while working in steps 6 and 7.

6. To move a label to a new position, position the Text tool over the label, not on the object itself. Click on the letter **B** in your sketch. It should look like the figure on the right when you hold down the mouse button. Drag the **B** around. Notice that you can move the label, but you cannot move it very far from the object it names. Try dragging other labels in your sketch.

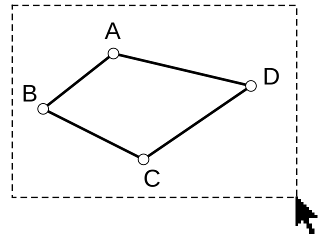
7. To rename a label, double click on the label you wish to change. Double click on the letter **G**. You should see the dialog box shown at right.

   Type **P** to replace the letter **G**. Click **OK.** The new name for the center of your circle should appear.

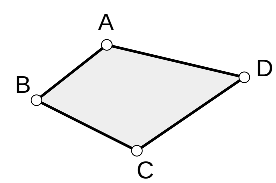
Relabel Dialog Box (Macintosh)

Use the Selection Arrow tool while working in steps 8–21.

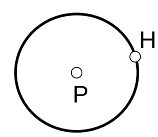
8. You’ve already seen how you can select an object by clicking on it. Also, if you hold the down Shift key, you can select several objects by clicking on them. Now you will learn another way to select one or more objects. Position the arrow in a blank area above and to the left of the quadrilateral. Drag down and to the right so that a dashed box surrounds the quadrilateral.



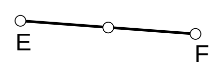
9. Release the mouse button. All objects inside the box should be selected. This method of selecting objects is known as “using a selection marquee.”
10. While it is still selected, drag any part of the quadrilateral. Notice that when all its parts are selected, you can move it without changing its shape.
11. Use the Selection Arrow tool to click in any blank space to deselect all objects.
12. To construct a polygon interior, hold down the Shift key. Click on the vertex points **A, B, C,** and **D** in clockwise or counter-clockwise order, then release the Shift key. Go to the **Construct** menu and choose **Polygon Interior.** While it is selected, you may change the shade or color of the interior by going to the **Display** menu and choosing **Shade** or **Color.**



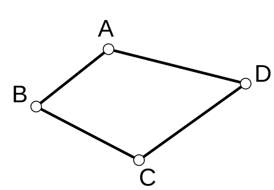
13. To construct the circle interior, click on the circle (not on one of the points on the circle). Go to the **Construct** menu and choose **Circle Interior.** You may use the **Display** menu to change the shade of the circle interior if desired.

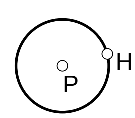


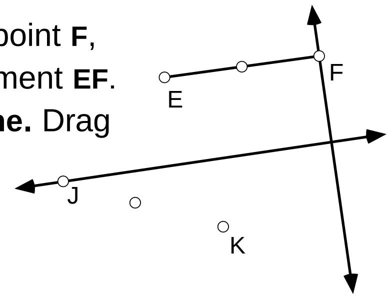
14. To construct the midpoint of a line segment, click on segment **EF**. Go to the **Construct** menu and choose **Point At Midpoint.** Drag point **F** and observe that the midpoint remains a midpoint regardless of the change in length of the segment.



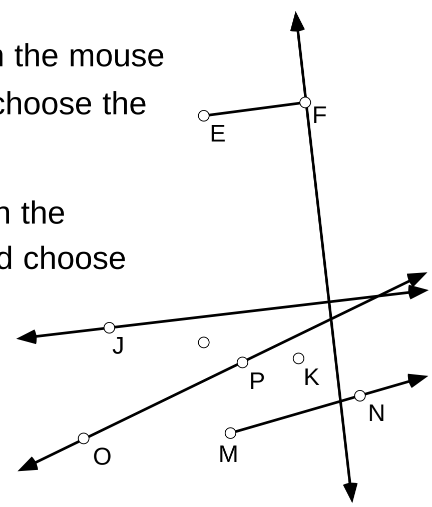
15. To hide segment **JK,** click on segment **JK,** go to the **Display** menu, and choose **Hide Segment.** The points **J** and **K** should remain even though the segment is hidden.
16. To hide several objects at once, you may use a selection marquee. Use the Selection Arrow tool to drag a box around quadrilateral **ABCD** and circle **P**. Go to the **Display** menu and choose **Hide Objects.**



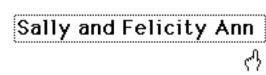




17. To construct a line perpendicular to segment **EF** through point **F**, hold down the Shift key. Click on point **F** and then on segment **EF**. Go to the **Construct** menu and choose **Perpendicular Line.** Drag point **E**. The line remains perpendicular.
18. Use the Selection Arrow tool to click in any blank space to deselect all objects.
19. To construct a line parallel to segment **EF** through point **J**, hold down the Shift key. Click on point **J** and then on $\overline{\textbf{EF}}$. Go to the **Construct** menu and choose **Parallel Line.** Drag point **E**. The lines remain parallel.



20. To construct a ray in your sketch, press and hold down the mouse button on the Segment tool. Drag to the right and choose the Ray tool. Drag to draw a ray in your sketch.
21. To construct a line in your sketch, press and hold down the mouse button on the Ray tool. Drag to the right and choose the Line tool. Drag to draw a line in your sketch.
22. Click on the Selection Arrow tool. Drag one of the control points that appeared on your ray or line. You should be able to change the direction of the line or ray when you drag.
23. Click on the Text tool and drag in a blank area at the top of your screen to make a text box.
24. Type your name inside the box.



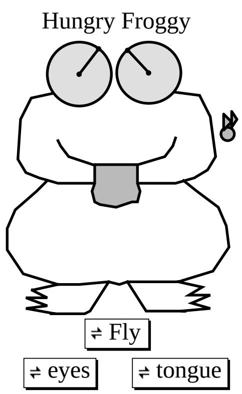
Summary

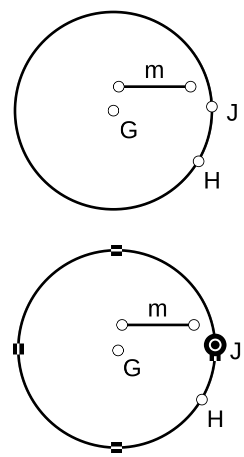
In this tour, you have learned to

- Construct a segment between two points using the **Construct** menu
- Construct a point on an object
- Construct the midpoint of a segment
- Construct rays and lines
- Construct perpendicular and parallel lines
- Construct circle and polygon interiors
- Move labels and change the name of a label
- Select objects by using a selection marquee
- Hide objects

Animation—Tour 1

An exciting feature of Sketchpad is animation. Animation in Sketchpad is the ability to give movement to constructed objects. You will discover many ways that animation will add a new dimension to your sketches as you explore. In this tour, you'll begin to learn how to create animations in Sketchpad.





1. Open the sketch Animation Tour (Mac) or Animtour.gsp (Windows).

First, you will animate a point on a circle.

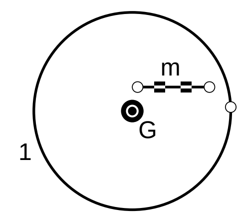
2. Notice that there are two points on the circle. Drag point **H** and observe that it is one of the control points of the circle. It changes the size of the circle. You cannot animate it on the circle for which it is a control point.
3. Drag point **J**. It was constructed as a **Point On Object.** It moves freely around the circle but does not change the size of the circle. This point can be animated about the circle.
4. Hold down the Shift key and, using the Selection Arrow tool, click on point **J**. Then click on the circle.
5. Go to the **Edit** menu and drag down to **Action Button.** Then, from the submenu, choose **Animation.**
6. When the **Path Match** dialog box appears, click on **Animate.** (You can experiment with different speeds and directions later.)
7. You should see an Animate button [Animate] appear on your screen. Double click on it. You will see point **J** move around the circle.

8. Click once to stop the animation. Click again in any blank space to deselect all objects.

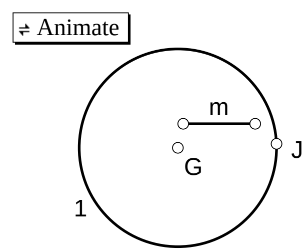
Relabel Dialog Box (Windows)

9. To change the button name, click on the Text tool, then double click on the Animate button. When the dialog box shown at right appears on your screen, type “Point on Circle.” Click on **OK.**
10. Use the Selection Arrow tool to click in any blank space to deselect all objects.

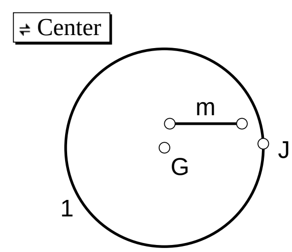
Next, you will animate a point along a segment.



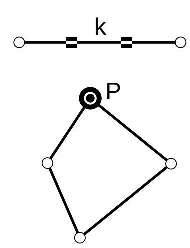
11. Click on the Selection Arrow tool. Hold down the Shift key. Click on point **G** and then on segment **m**.
12. Go to the **Edit** menu and choose **Action Button.** Choose **Animate** from the submenu.
13. When the **Path Match** dialog box appears, click on Animate.



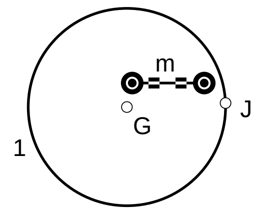
14. You should see an Animate button appear on your screen. Double click on the button. You will see point **G** move along segment **m**. Click once to stop the animation. Experiment by changing the length and location of the segment and animating again.



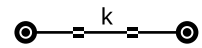
15. To change the button name, click on the Text tool, then double click on the Animate button. When the **Relabel** dialog box appears on your screen, type Center. Click on **OK.**
16. Use the Selection Arrow tool to click in any blank space to deselect all objects.



17. Hold down the Shift key. Click on point **P** and then on segment **k**.
18. Go to the **Edit** menu and choose **Action Button,** then choose **Animation** in the submenu. Click **Animate** in the **Path Match** dialog box.
19. You should see an Animate button appear on your screen. Double click on the button. You should see the point **P** move along segment **k**.
20. Change the name of the button if you wish. (See step 15 above.)
21. Use the Selection Arrow tool to click in any blank space to deselect all objects.

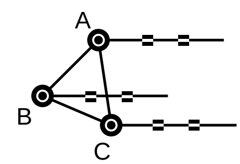


22. You may hide unwanted objects even if they are used as part of the animation. For example, hold down the Shift key and click on segment **m** and both its endpoints. Go to the **Display** menu and choose **Hide Objects.** Double click on the Center animation button again. The animation should occur even though the segment is no longer showing.



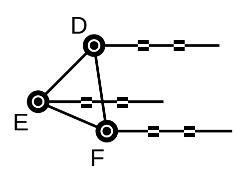
23. Follow step 22 to hide segment **k** and its endpoints.

Now you will use animation techniques with a polygon.



24. To move an entire shape, you may need to animate points on congruent segments, as with triangle **ABC**. To do this, hold down the Shift key, click on point **A,** and then click on the segment to its right. Click on point **B** and the segment to its right. Then click on point **C** and the segment to its right.
25. Go to the **Edit** menu and choose **Action Button,** then choose **Animation** in the submenu. When the **Path Match** dialog box appears, click on **Animate.**
26. Double click on the Animate button. You should see triangle **ABC** move across the screen. Hide the path segments and change the name of the button if you wish.
27. Use the Selection Arrow tool  to click in any blank space to deselect all objects.

Now you will explore using different speeds in animation.



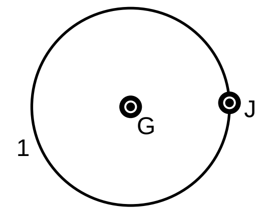
28. Hold down the Shift key and click on point **D** and the segment to its right. Click on point **E** and the segment to its right. Finally, click on point **F** and the segment to its right.
29. Go to the **Edit** menu and choose **Action Button,** then choose **Animation** in the submenu. When the **Path Match** dialog box appears, each of the three animations will be set to go bidirectionally along a segment quickly. Click on the second statement in the list. Press the pop-up menu that says “quickly” and change it to “normally.” Click on the third statement and use the pop-up menu to change “quickly” to “slowly.”

Animation Path Match Dialog Box (Macintosh)

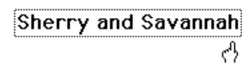
30. Double click on the Animate button. You should see triangle **DEF** move across the screen. Hide the segments and change the name of the button if you wish.

Next, you will trace a locus, a process closely related to animation.

31. Hold down the Shift key. Click on points **G** and **J**. Go to the **Construct** menu and choose **Segment.**



32. While the segment is selected, go to the **Display** menu and choose **Trace Segment.** Now double click on the Point on Circle animation button again. What you see is the path of $\overline{GJ}$ as point **J** moves around the circle.
33. Experiment with animation on your own.
34. Click on the Text tool  and drag a box at the top of your screen.



35. Type your name inside the box.

Summary

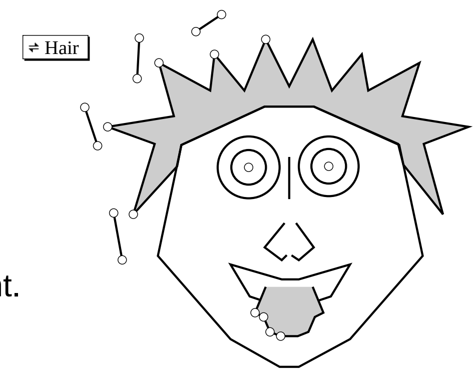
In this tour, you have learned to

- Animate a point on a circle
- Animate a point on a segment
- Animate a polygon's vertices on congruent, parallel segments at the same speed so that the polygon moves and its shape is preserved
- Animate objects at different speeds
- Change the name of an animation button
- Trace a locus

Animation—Tour 2

In this tour, you will learn more techniques of animation in Sketchpad.

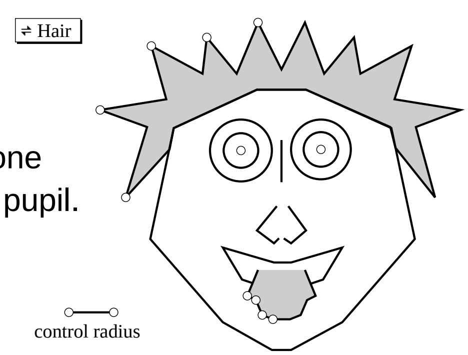
1. Open the sketch Jack (Mac) or Jack.gsp (Windows). One side of Jack's face was constructed, and then that half was reflected across a vertical mirror to complete his face. Therefore, when you animate an object on one side of the face, the animation will occur on both sides.



Follow steps 2–8 to animate Jack's hair.

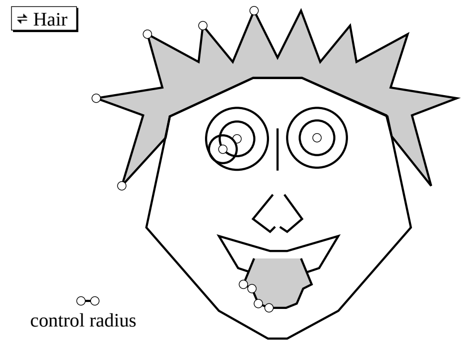
2. To animate Jack's hair, use the Segment tool to construct short segments close to Jack's hair. Your sketch should look similar to the drawing at the right.
3. Click on the Selection Arrow tool. Click in a blank space to deselect objects.
4. Hold down the Shift key. Select a point on the hair and then select the segment (not its endpoints) closest to that point. Repeat until you have selected all the points of the hair and the segments closest to them.
5. Go to the **Edit** menu and choose **Action Button.** Choose **Animate** in the submenu.
6. An Animate button should appear in your sketch. Use the Text tool to change the name of the button if desired. Double click on the Animate button to see the animation.
7. Click once to stop the animation, then click again in a blank space to deselect objects.
8. To hide the segments and their endpoints, hold down the Shift key. Click on the segments and their endpoints until you have selected them all. Go to the **Display** menu and choose **Hide Objects.**

Follow steps 9–21 to construct animated pupils in Jack's eyes.

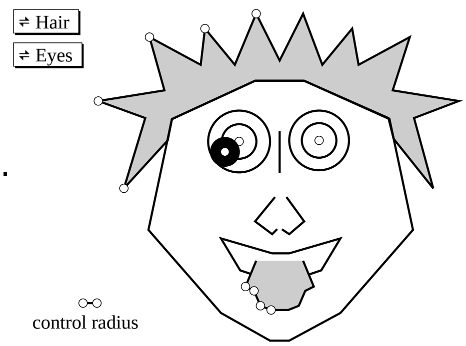


9. Click on the Segment tool.
10. Draw a small segment in your sketch similar to the one shown at right. This will be the control radius for the pupil.
11. Use the Selection Arrow tool to click on the inner circle in one eye (not the center point). Go to the **Construct** menu and choose **Point On Object.**

12. Click in a blank space to deselect objects.
13. Hold down the Shift key. Click on the point that appeared on your small circle and then click on the control radius segment.



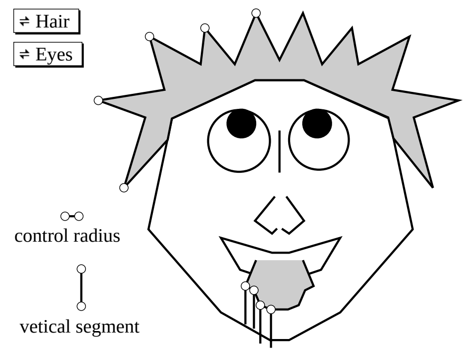
14. Go to the **Construct** menu and choose **Circle By Center+Radius.** Drag an endpoint of your control radius segment to change the size of the pupil so that it fits inside the eye.
15. To animate this pupil inside the eyeball, use the Selection Arrow tool. Click in a blank space to deselect objects. Hold down the Shift key. Click on the center point of the new circle and then click on the middle circle.



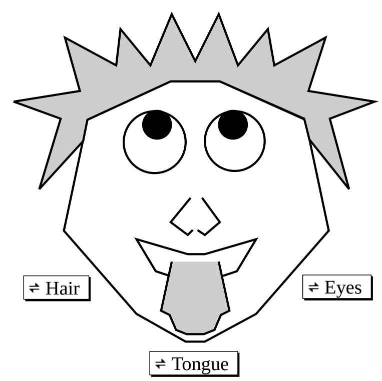
16. Go to the **Edit** menu and choose **Action Button.** Choose **Animate** from the submenu. Use the Text tool to change the name of the button if you like.
17. Use the Selection Arrow tool. Click in a blank space to deselect objects.
18. To fill in the pupil, click on the pupil circle. Go to the **Construct** menu and choose **Circle Interior.**
19. Hide the middle circle and the center point of the eyes. To do this, use the Selection Arrow tool. Click in a blank space to deselect objects. Hold down the Shift key. Click on the middle-sized circles and the center points of both eyes. Go to the **Display** menu and choose **Hide Objects.**
20. Click on the vertical segment between the eyes. Go to the **Transform** menu and choose **Mark Mirror.**
21. Click on the pupil. Go to the **Transform** menu and choose **Reflect.** Double click on your Animate button to see the animation.

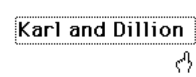
Follow steps 22–31 to animate Jack's tongue.

22. To animate the tongue up and down, use the Segment tool to construct a short vertical segment.
23. While the vertical segment is selected, go to the **Edit** menu and choose **Copy.**
24. Go to the **Edit** menu and choose **Paste.** Paste three more times so that you have four copies of the vertical segment pasted in your sketch.



25. Drag each vertical segment directly below a different one of the four points on one side of Jack's tongue.
26. Use the Selection Arrow tool. Click in a blank space to deselect objects.
27. Hold down the Shift key. Click on a point on the tongue and then click on the vertical segment directly under that point. Continue to do this until you have all four points and the vertical segments below them selected.
28. Go to the **Edit** menu and choose **Action Button.** Choose **Animate** in the submenu. Use the Text tool to change the name of the button if you desire.
29. Use the Selection Arrow tool. Hold down the Shift key and click on the vertical segments. Go to the **Display** menu and choose **Hide Segments.**
30. Hide any other unwanted segments and points.
31. Double click on the animate buttons to watch the animation. What a lovely Jack!
32. Experiment on your own.
33. Click on the Text tool and drag a box at the top of your screen.
34. Type your name inside the box.





Summary

In this tour, you have learned to

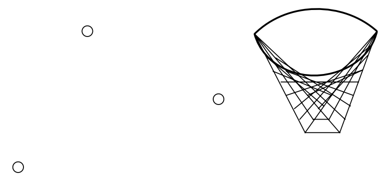
- Animate a point on a polygon (the hair) and on a segment to move part of the polygon
- Construct a circle (the pupil of the eye) using a control radius segment and animate the center of the circle around another circle
- Animate a polygon (the tongue) on congruent segments at the same speed so that the polygon moves and its shape is preserved
- Reflect an object (the pupil) about a mirror

Animation—Tour 3

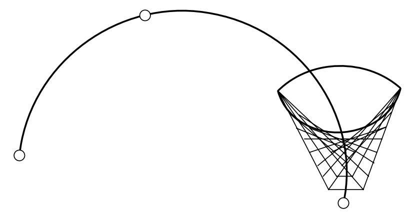
In this tour, you will learn more techniques of animation in Sketchpad.

1. Open the sketch Basket (Mac) or Basket.gsp (Windows).

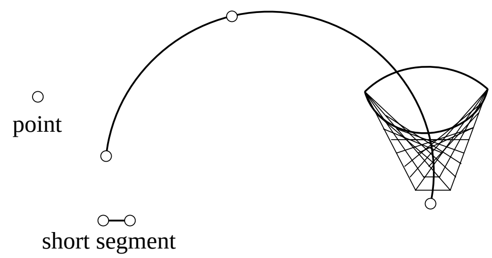
Follow steps 2–13 to construct a ball and animate it through the basket.



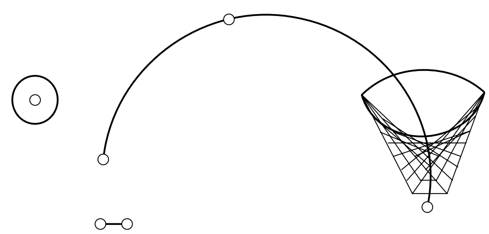
2. First, use the Point tool to construct three points in blank space to the left of the basket.
3. Click on the Selection Arrow tool. Select the three points. Go to the **Construct** menu and choose **Arc Through 3 Points.**



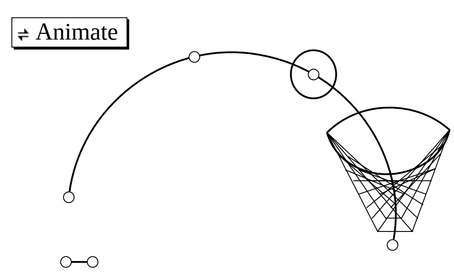
4. Drag one of the endpoints of your arc until you have the arc positioned so that it appears to pass through the basket.
5. Use the Segment tool to construct a short segment somewhere in the blank area of your sketch.



6. Use the Point tool to construct a point somewhere in the blank area of the sketch.
7. Use the Selection Arrow tool to click in a blank space to deselect objects. Hold down the Shift key and select both the point and the short segment. (Remember to click on the segment, not on its endpoints.)

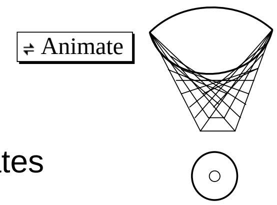


8. Go to the **Construct** menu and choose **Circle By Center+Radius.** Drag an endpoint of the short segment to change the size of the ball.



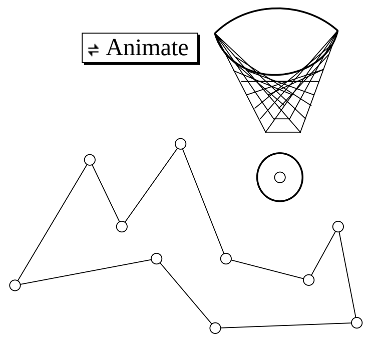
9. Use the Selection Arrow tool to click in a blank space to deselect objects. Hold down the Shift key and click on the center point of the ball and then click on the arc to select it. (Do not click on the points on the arc.)
10. Go to the **Edit** menu and choose **Action Button.** Choose **Animation** from the submenu. Change the direction to "one way." Then click on **Animate.**

11. Double click on the new Animate button to see your animation.
12. Click once to stop the animation. Then click again in a blank space with the Selection Arrow tool to deselect objects. Hold down the Shift key and click on the arc and its three defining points and on the short segment and its endpoints.

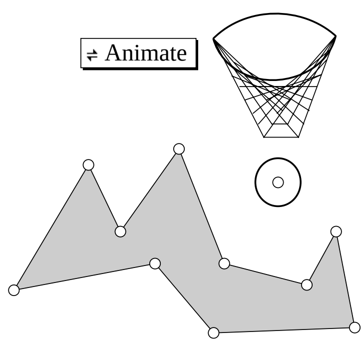


13. Go to the **Display** menu and choose **Hide Objects.** Double click on the Animate button again. Notice that the ball still animates along the arc even though the arc is hidden.

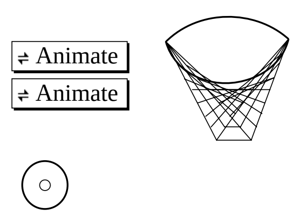
Follow steps 14–24 to create a bouncing ball in your sketch.



14. Use the Segment tool to construct a many-sided polygon below the basket.
15. Use the Selection Arrow tool to click in a blank space to deselect objects.
16. Hold down the Shift key. Select the vertex points of your polygon in clockwise or counterclockwise order.

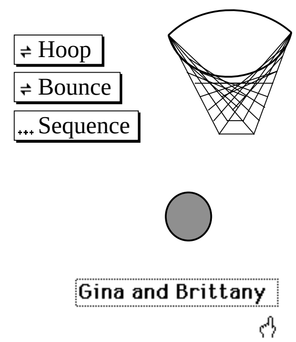


17. Go to the **Construct** menu and choose **Polygon Interior.**
18. Use the Selection Arrow tool to click in a blank space to deselect objects.
19. Hold down the Shift key. Click on the center point of the ball and click on the interior of the polygon.
20. Go to the **Edit** menu and choose **Action Button.** Choose **Animation** in the submenu. Change directions if you desire. Click on **Animate.**



21. Double click on this new Animate button. Watch your ball bounce around the polygon. Click once to stop the animation.
22. Drag your ball away from the polygon. Use the Selection Arrow tool to select the polygon interior and all the sides and points that define the polygon. Go to the **Display** menu and choose **Hide Objects.**
23. Double click on either of your Animate buttons and watch your animation.

24. You can construct the circle interior and hide the center point of the ball if you desire.
25. Use the Text tool to change the name of the buttons if you desire.
26. Use the Selection Arrow tool to click in a blank space to deselect objects.
27. Select both Animate buttons. Go to the **Edit** menu and choose **Action Button.** Choose **Sequence** in the submenu.
28. Double click on the Sequence button to start the first animation. Click once to stop the first animation and start the second animation.



29. Experiment on your own!
30. Click on the Text tool and drag a box at the top of your screen.
31. Type your name inside the box.

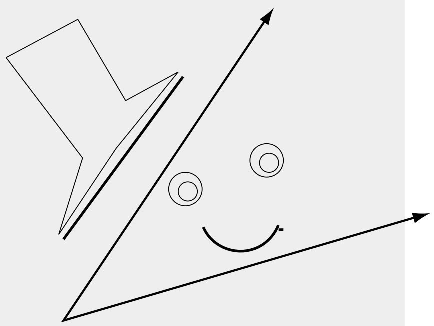
Summary

In this tour, you have learned to

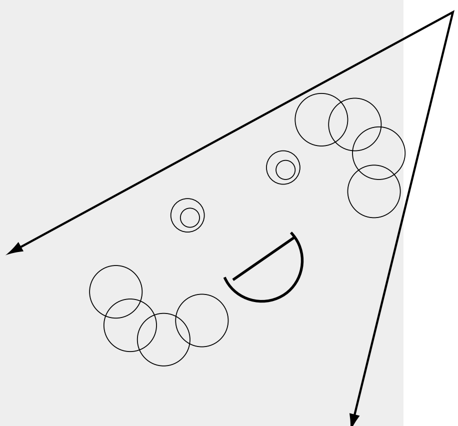
- Construct an arc and animate a point (the ball's center) along the arc
- Animate a point (the ball's center) along a polygon interior
- Set up a Sequence button

Unit 2

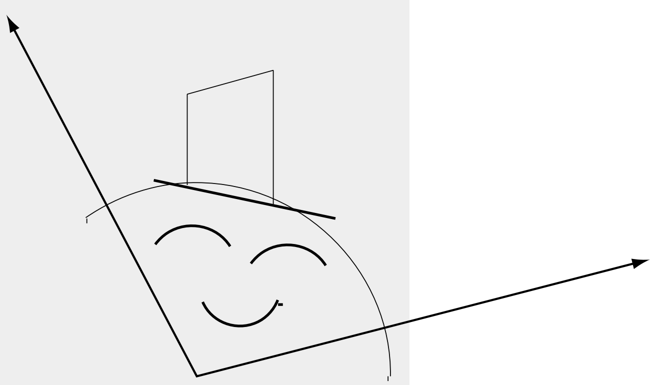
POINTS, LINES, AND ANGLES



Kyle

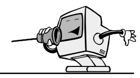


Ashley



Clint

Commentary



This unit is especially suitable for students who are encountering for the first time the concepts about lines and angles presented in the activities. Formulating their own definitions or lists of characteristics for these concepts not only gives the students ownership of their work, but also enables them to remember the concepts much better.

For many groups of students, it will be appropriate to present some of the activities in this unit as teacher- or student-led demonstrations rather than as independent explorations by each child. This is true both for groups first encountering these topics mathematically and for groups who are already familiar with the topics but who need a quick review. Of course, you must decide which activities and formats best meet the skill levels and needs of a particular group.

Pesky Points helps students understand how points function in the Sketchpad program. This introduction is very important for all students, regardless of their mathematical background, unless they have previous experience with Sketchpad. This is one of the activities that may best serve your students' needs when done as a teacher demonstration; in this case, you can use the student activity sheet as a checklist to ensure that you explain the ways points function with Sketchpad. You may want the students to choose **Auto Show Label for Points** in **Preferences** under the **Display** menu as they begin this activity. They should understand the following characteristics of points:

- Points constructed as free points can be dragged anywhere in the sketch.
- A point constructed at the intersection of two objects may drag the objects for which it is an intersection point.
- A point constructed as a **Point On Object** can be dragged anywhere along or around the object but will not drag the object itself and cannot be dragged off the object.
- When you use the Segment, Ray, Line, or Circle tool, two control points appear as you construct these objects. These points will change the orientation of the object and, in the case of the segment and circle, will also control the size of the object on which they are constructed.

A Point of Interest is designed to lead students to discover that there are an infinite number of points equidistant from two given points. This activity also serves as an exploratory introduction to tracing a locus of a point and requires very little technical expertise.

Since the study of points, lines, and angles usually appears early in the middle-school geometry curriculum, most students doing the three **Exploring Angles** activities will be new to Sketchpad. Because of this, the teacher demonstration needs to be done slowly and thoroughly and may require repetition or additional examples for some groups of students. Keep in mind that, for many beginners in dealing with angles, the orientation of an angle is a stumbling block unless this characteristic is addressed very early. Students readily accept that an angle can "face any direction" unless they have measured a large number of angles all oriented in the same direction before they are exposed to one turned a different way. Be sure that students drag to form the angles listed in each section of the activity. Repeating the process helps develop confidence in their Sketchpad skills as well as improve their memory for how to perform the maneuver.

In **Exploring Angles,** the sketch Angles 2 is not dynamic, so students can stay focused on target concepts. At this beginning level, the sketch provides practice in visual recognition of types of angles and in using Sketchpad.

For **Exploring Special Pairs of Angles,** we have found that the most important things to keep in mind during the teacher demonstration are essentially the same as those for **Exploring Angles.** Remembering that the students are Sketchpad beginners as well as newcomers to the mathematical concepts helps ensure the success of the activity. The continual updating of measures by Sketchpad provides much of the impact of this demonstration. In our classroom, this feature of Sketchpad has convinced more students that some things simply are *always* true than has the text or all our proclamations! You will have allies: Listen for one student to say to another, “It must happen all the time. We’ve tried all the ways there are, and it always turns out the same. Don’t you see those numbers on the screen?”

Exploring Angles Formed by Parallel Lines and a Transversal is one of the activities that most clearly demonstrates the power of Sketchpad. So many examples appear in such a short time that students quickly grasp a concept that is traditionally time-consuming and tricky to teach. Students repeatedly refer to this activity later in the year: “Don’t you remember? Those angles are always congruent, like we did with Sketchpad when we dragged that transversal, you remember.”

Interesting Angles has often surprised us. Students who we suspected might have trouble have often enjoyed this project the most: kids see it as a puzzle. The inevitable experimentation builds confidence with Sketchpad, and kids are often eager to show off unique solutions. Promoting “illustrations” like those at the top of the instructions adds to the fun. If your students wish to use arrows, you may need to load the Script tool. Directions are included in Instructions for Teacher Demonstrations later in these notes.

Name Angles has been a great motivator for our kids. They love to show their creations to the class. Having classmates guess the measure of the angles in each of the sketches, followed up by measuring with Sketchpad or a protractor, is a fun way to practice, too.

Angle Puzzle is considerably more difficult than the activities. Most likely, it will be used best as a reinforcement or extension at a later time or with more knowledgeable groups. Certainly, **Angle Puzzle** should be done in groups, since students working alone may get frustrated with it. It is an especially good activity when you are trying to establish the value of cooperative learning, because different individuals invariably notice different details that help unravel the “mystery” for the group. We have found it effective to let students work alone at first, then to pair them with a partner after a short time, and finally to combine two pairs a little later.

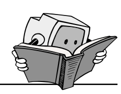
Angle Puzzle is not dynamic and, in fact, does not require the use of Sketchpad at all; however, we included it because it involves valuable synthesis and application. It also provides an alternative assessment instrument that requires substantial application of the concepts presented in the activities in this unit. The Sketchpad component, of course, is the Challenge at the end. Having students work each other’s Challenges is a must; furthermore, students should view their first attempts only as a draft, and they should gain insight from initial errors.

Students should be warned to read the directions for **Sharp Shooter** extremely carefully. The kids to whom this project appeals especially enjoy the portion in which the ball must bounce off two walls, but it is too difficult for some middle-schoolers. Knowledge of reflections enhances this project and suggests alternative solutions.

The Sketchpad skills required by **Creating a Sketchpad Clock** may cause you to deem it more appropriate at some later point in the year. For some reason, the notion of creating an accurate model of a clock comes up with almost every new group of students who work with Sketchpad, but they really need knowledge of animation, reflection, dilation, and rotation before they can successfully create a “working” model. Students will persevere and take great pride in demonstrating their final product for their classmates. This project is a terrific “teaser” to mention early in the year and revisit after students have done transformations. Of course, some kids could explore transformations on their own using the activities in Unit 6 and then try the project if you feel it is appropriate.

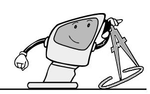
Referring to the Wrap-Up for this unit, we would like to emphasize an observation: While for some middle-school kids the *always, sometimes, never* format is a little too difficult, our experience is that many kids improve dramatically after practice with this format and that the understanding gained is clearly more than that accomplished by true-false questions. We would encourage you to try the format several times before becoming discouraged.

Prerequisite Mathematical Terms and Concepts

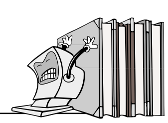


- *point, line, ray, segment, endpoint*
- measuring angles

Recommended Sketchpad Proficiency



- Basic knowledge of the freehand tools; beginner-level Sketchpad proficiency.
- Use of the Points, Lines, and Angles Quick Reference Guide (found at the end of this unit) is suggested.
- Doing Tours 1 and 2 would be beneficial (see Tours unit).



Essential Vocabulary

Acute angle—any angle measuring less than 90° and greater than 0°

Alternate exterior angles—a pair of angles formed by parallel lines and a transversal where both angles are outside the parallel lines but on opposite sides of the transversal

Alternate interior angles—a pair of angles formed by parallel lines and a transversal where both angles are inside the parallel lines but on opposite sides of the transversal

Angle—two rays with a common endpoint; usually named by three letters with the vertex label as the middle letter

Complementary angles—a pair of angles whose measures total 90°

Corresponding angles—a pair of angles formed by parallel lines and a transversal where one angle is outside the parallel lines and the other is inside, and both angles are on the same side of the transversal

Obtuse angle—any angle measuring greater than 90° and less than 180°

Parallel lines—two or more lines in a plane that never intersect

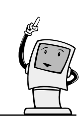
Right angle—any angle measuring exactly 90°

Straight angle—any angle measuring exactly 180°

Supplementary angles—a pair of angles whose measures total 180°

Transversal—a line that intersects two lines to form eight angles

Vertical angles—a pair of angles that are formed by a pair of intersecting lines and do not share a side



Instructions for Teacher Demonstrations

In each unit, we have included sketches and guidelines for teacher demonstrations designed as introductions by the teacher in whole-class presentations before students attempt the various activities independently. They include the introduction of relevant mathematical vocabulary and concepts as well as appropriate Sketchpad skills. Often there are one or two activities that you will feel comfortable assigning to your students with very little guidance beforehand. In this unit, for example, **A Point of Interest** works fine as an exploratory introduction to tracing a locus of a point and requires very little technical expertise. On the other hand, one or more activities in each unit are designed to maximize guided discovery by the students. For these activities, a careful teacher demonstration will focus the students' attention on targeted mathematical concepts and prevent distractions due to lack of Sketchpad proficiency. In this unit, the activities **Exploring Angles, Exploring Special Pairs of Angles,** and **Exploring Angles Formed by Parallel Lines and a Transversal** are designed to follow such a demonstration. Instructions are included later in these notes. Of course, you may want to use portions or adaptations of the demonstrations before other activities; certainly, you will want to preview activities to be sure they are appropriate for the mathematical and Sketchpad skill levels of your students. Even with careful demonstrations, many students will forget one or more steps when they try something for the first time, for example, measuring an angle at the com-puter. You need to show several examples and then, at the close of the teacher demonstration, have a student sit at the computer while another student gives verbal instructions.

We find it very valuable to solicit student suggestions during the demonstrations. In the parallel lines demonstrations, for example, a student invariably suggests first using the freehand Segment or Line tool to construct a line parallel to a given line. Errors like this stimulate thought, result in lively class discussion, and provide many opportunities for guided discovery.

For some of the activities in this unit, you and your students may want to use the Sample Utility Scripts as Script tools. To load the Utility Scripts as Script tools, do the following steps:

1. Go to the **Display** menu and choose **Preferences.**
2. Click on **More.**
3. Click on **Set** in the Advanced Preferences dialog box where it says **Script Tool Folder.** (You may have to click **Clear** first.)
4. Open the Sample Scripts folder in The Geometer's Sketchpad folder.
5. Open the Utility Scripts folder (Mac) or the Utilities directory (Windows).
6. Click on **Select "Utility Scripts".**
7. Click on **Continue.**
8. Click on **OK.** You should see the Script tool added to the bottom of your toolbox.

You may find the Points, Lines, and Angles Quick Reference Guide (at the end of this unit) helpful in preparing for the teacher demonstrations.

Exploring Angles

Introducing Points, Segments, Rays, and Lines

Open a new sketch. Use the freehand tools to demonstrate and discuss *point*, *segment*, *ray*, and *line.*

- Construct free points.
- Construct and drag a segment. (Discuss endpoints.)
- Construct and drag a ray. (Discuss the endpoint and other control point.)
- Construct and drag a line. (Discuss the two control points.)

Introducing Angles

1. Open a new sketch. Use the Segment and Ray tools to construct several angles. Demonstrate the following Sketchpad skills.
    - Construct an angle using the Segment tool.
    - Construct an angle using the Ray tool.
    - Measure both angles. (Be sure you orient the angles in several different directions, for example, as and . Also emphasize that the vertex must be the middle point selected when measuring.)

2. Point out to students that angles in some sketches in this unit will appear as "textbook angles," with arrows at the ends of segments to represent rays. Others will appear with rays that go off the sketch, implying that the rays continue indefinitely in one direction.
3. Use the same sketch to show and discuss acute, obtuse, and right angles.
4. Using PLA Demo 1 (Mac) or Plademo1.gsp (Windows), discuss and demonstrate the interior and exterior of an angle. Discuss and construct points on the angle, in the interior of the angle, and in the exterior of the angle.

Exploring Special Pairs of Angles

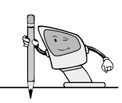
1. Review *angles* and naming angles.
2. Demonstrate the following Sketchpad skills using PLA Demo 2 (Mac) or Plademo2.gsp (Windows).
   - Drag point **A**, **B**, **C**, or **D** using the Selection Arrow tool. Each of these points changes the angle measure or orientation of the angle when dragged.
   - Drag any of the points at the end of the arrows to show that the measure of the angle does not depend on the length of the indicated ray. (Remind students that the ray should be thought of as having infinite length.)
   - Make observations from displayed measures (e.g., m∠**ABD** + m∠**DBC** always equals m∠**ABC**).
3. Discuss the angles formed by intersecting lines. Demonstrate the following Sketchpad skills using PLA Demo 3 (Mac) or Plademo3.gsp (Windows).
   - Display the measure of any angle. Do not display the measures of all four angles, since students will be doing this in the activity.
   - Drag point **S** or **B** to change the measures of the angles.

Exploring Angles Formed by Parallel Lines and a Transversal

1. Discuss the angles formed by a transversal intersecting two parallel lines. Demonstrate the following concepts using PLA Demo 4 (Mac) or Plademo4.gsp (Windows): *transversal, corresponding angles, alternate interior angles,* and *alternate exterior angles.*
2. Discuss the relationship between the various angles formed using PLA Demo 4.
   - Measure some of the angles within the figure.
   - Demonstrate how the orientation of the lines and the measures of the angles change as you drag point **A**, **B**, or **D**.
   - Demonstrate how the orientation of the transversal and the measures of the angles change as you drag point **N**.

Note: For some classes, it will be more appropriate to do **Exploring Angles Formed by Parallel Lines and a Transversal** totally as a teacher demonstration. If you do so, you may want to demonstrate not only with PLA Demo 4 but also with the sketch Parallel (Mac) or Parallel.gsp (Windows).

Writing Prompts

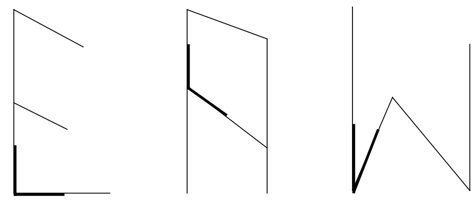


Ask students to choose a topic below and to write a short paper, including sketches created with Sketchpad.

- Angles Party (and the Special Pairs Who Attend)
- Important Points About Angles
- Points, Lines, and Angles—The Basics of Geometry
- Taking a Geometric Walk (Observing Lines and Angles in My Neighborhood)

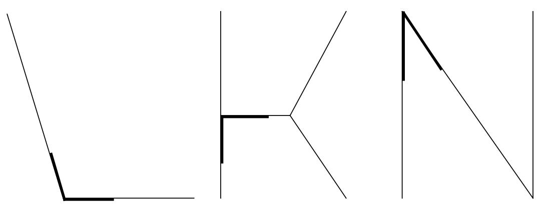
Examples of Student Work

Name Angles Project	Interesting Angles Project



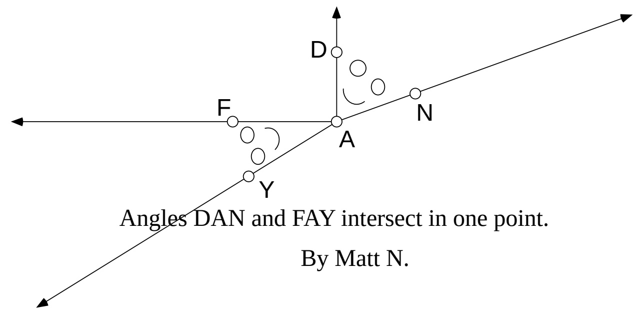
Emily Ann Wyatt

m∠E = 90° m∠A = 127° m∠W = 23°



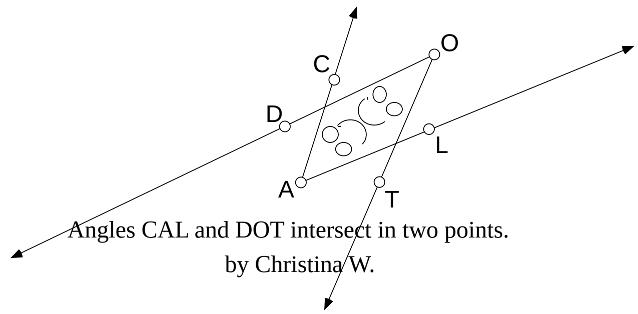
Laura Kathryn Nolin

m∠L = 107° m∠K = 90° m∠N = 35°



Angles DAN and FAY intersect in one point.

By Matt N.



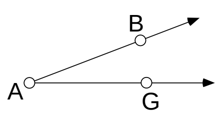
Angles CAL and DOT intersect in two points.

by Christina W.

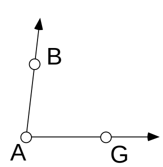
Answers for Exploring Angles Activity

Orientation of these angles will vary.

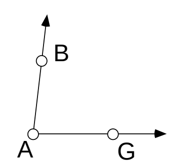
2. a.



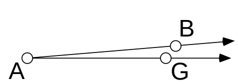
b.



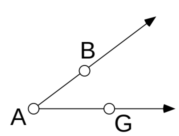
c.



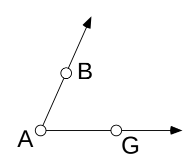
d.



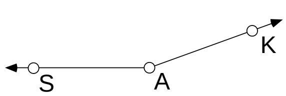
e.



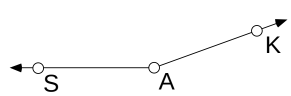
f.



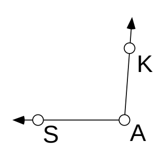
3. a.



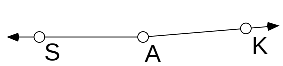
b.



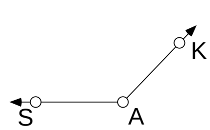
c.



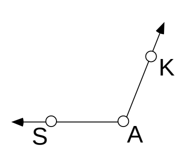
d.

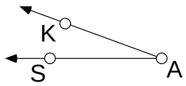


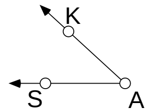
e.

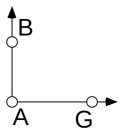


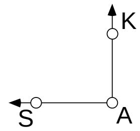
f.



4. a. 

   b. 

5. a. 

   b. 

7. acute, 70°
8. a. acute, 60° b. obtuse, 155° c. acute, 45°
   d. obtuse, 95° e. acute, 40° f. right, 90°
   g. obtuse, 135° h. right, 90° i. acute, 20°
   j. right, 90°

Answers for Exploring Special Pairs of Angles Activity

2. The sum equals 90°.
3. two angles whose sum is 90°
5. The sum equals 180°.
6. two angles whose sum is 180°
9. They are always equal.
10. They are always equal.
11. A pair of vertical angles have equal measures.
12. supplementary; supplementary
13. You need to know only one angle measure. If you know one angle, it is equal to one of the angles and supplementary to the other two angles.

Answers for Exploring Angles Formed by Parallel Lines and a Transversal Activity

2. They are equal in measure.
3. a. $\angle$**EOR**, $\angle$**HEM** = $\angle$**EOR** b. $\angle$**RON**, $\angle$**MEO** = $\angle$**RON**
   c. $\angle$**YON**, $\angle$**XEO** = $\angle$**YON**
4. equal; equal in measure
5. They are equal in measure.
6. $\angle$**YEO**; $\angle$**EOR**; Their sum is 180°.
7. They are equal; equal
8. They have equal measures.
9. $\angle$**HEX**; $\angle$**RON**; They are equal.
10. They are equal; equal in measure

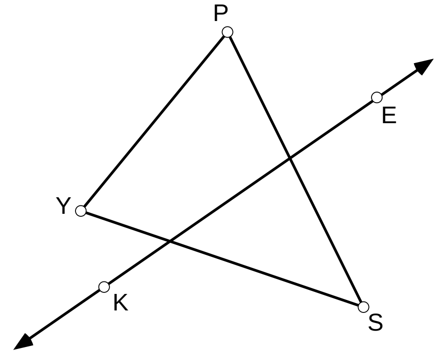
Answers for Angle Puzzle Project

m∠1 = 130°	m∠2 = 130°	m∠3 = 50°
m∠4 = 30°	m∠5 = 70°	m∠6 = 70°
m∠7 = 60°	m∠8 = 40°	
m∠9 = 40°	m∠10 = 50°	

Answers for Points, Lines, and Angles Wrap-Up

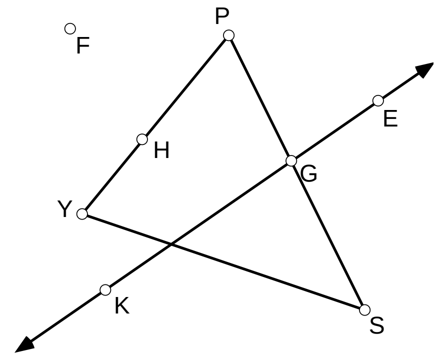
1. acute	2. obtuse	3. right
4. acute	5. obtuse	6. acute
7. a. supplementary	b. 155°	
8. a. complementary	b. 40°	
9. a. vertical	b. 135°	
10. a. vertical and complementary	b. 45°	
11. a. alternate interior	b. 40°	
12. a. corresponding	b. 130°	
13. a. alternate exterior	b. 140°	
14. a. corresponding	b. 60°	
15. sometimes	16. sometimes	17. sometimes
18. always	19. never	20. sometimes

Pesky Points



Because Sketchpad is a dynamic tool, points will behave differently depending on how they are constructed. This activity helps you explore such points.

1. Open the sketch Points (Mac) or Points.gsp (Windows).
2. Using the Point tool, click to create a point somewhere in the sketch so that it does not touch any of the objects already in the sketch. This point will probably be labeled point **F**. If the label does not appear when you create the point, click on the point using the Text tool.
3. Click on the Selection Arrow tool. Click in a blank space to deselect objects.
4. Hold down the Shift key and click on line **EK** and segment **PS** with the Selection Arrow tool to select them (don't click on the points on the line or the points on the segment). Go to the **Construct** menu and choose **Point At Intersection.** This new point will probably be labeled point **G**.



5. Click on the segment **PY** (not on the points **P** and **Y**). Go to the **Construct** menu and choose **Point On Object.** This new point will probably be labeled point **H**.

Your sketch probably looks similar to the one above. Now you will experiment with the way different points behave.

6. Use the Selection Arrow tool to drag point **F**. Describe the effects on the figure.

___

___

___

7. Use the Selection Arrow tool to drag point **H**. Describe the effects on the figure. Did points **F** and **H** behave in the same manner? Explain. ___

___

___

8. Use the Selection Arrow tool to drag point **G**. Describe the effects on the figure. Is the behavior of point **G** like that of point **F** or **H**? Explain. ______

9. Use the Selection Arrow tool to drag point **E**. Describe the effects on the figure. What happens to point **G** when you drag point **E** so that the line no longer crosses the triangle? ______

10. Explain why you think the different points behaved differently. ______

11. Open a new sketch.

12. Use the Circle tool to construct a circle in your sketch.

13. Use the Selection Arrow tool to drag the center of your circle. What happened to the circle? ______

14. Now drag the point on the circle. What happened? ______

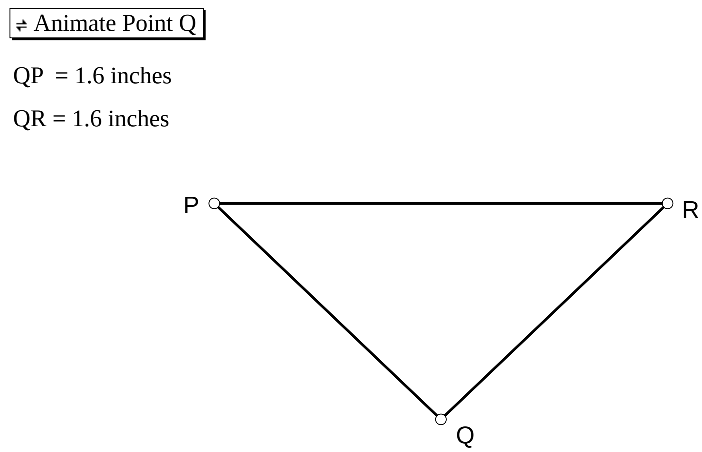
15. Click on the circle to select it (don't select the center point or the point on the circle). Go to the **Construct** menu and choose **Point On Object** to construct another point on the circle. Drag this point. Describe what happens. ______

16. Explain why you think the different points behaved differently. ______

A Point of Interest

If you are given two points, how many points are there that are equidistant (the same distance) from the two given points?

Because Sketchpad is a dynamic tool, points can be traced (can leave a path) when animated. This activity helps you explore such a point and answer the question above.

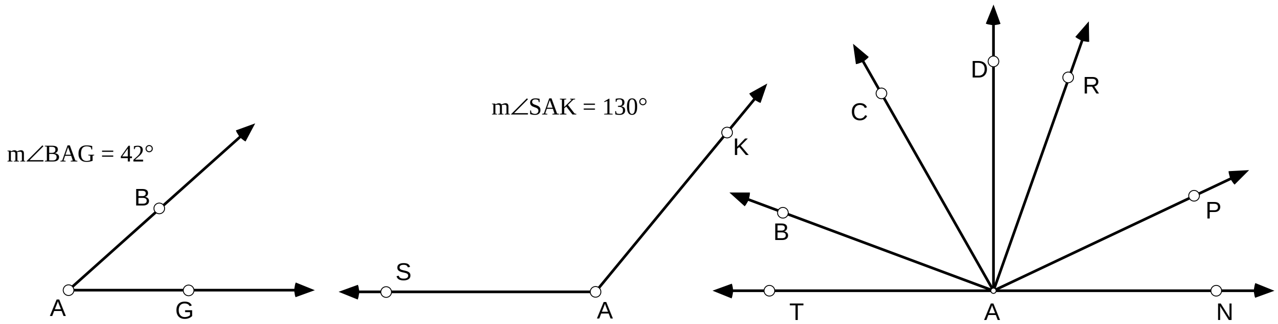


1. Open the sketch Equidistant (Mac) or Equidist.gsp (Windows). Notice that the distances between points **Q** and **P** and points **Q** and **R** are displayed in the sketch. As you can see, point **Q** is equidistant from points **P** and **R**.
2. Carefully drag point **Q** so that the distances change, but try to keep the distances as close to equal as you can. Is there more than one position where point **Q** is equidistant from points **P** and **R**? ______________________
3. Use the Selection Arrow tool and double click on Animate Point **Q** in your sketch to observe the path of point **Q**. Notice the distances as they are updated on your screen. Are there positions besides the one where point **Q** started that are equidistant from points **P** and **R**? ______________________
4. To stop the moving point, click with the mouse *once.* Describe the path that point **Q** seems to trace. ______________________
5. Based on your observations, answer the question at the beginning of this activity.

____

____

Exploring Angles



Two rays with a common endpoint form an angle. Use Sketchpad to explore angles.

1. Open the sketch **Angles 1** (Mac) or **Angles1.gsp** (Windows).
2. Notice ∠**BAG**. It is an *acute angle* (an angle with measure less than 90°). For each measure listed below, drag either **B** or **G** to observe the angle and make a sketch of your result.

   a. 20° b. 45° c. 82.5°

   d. 5° e. 35.5° f. 65°

3. Notice ∠**SAK**. It is an *obtuse angle* (an angle with measure between 90° and 180°). For each measure listed below, drag **S** or **K** to observe the angle and make a sketch of your result.

   a. 120° b. 160° c. 95°

   d. 175.4° e. 135° f. 110.7°

4. Of course, you can drag ∠**SAK** to make it an acute angle as well. Drag **S** or **K** to observe an angle with each measure listed below and make a sketch of your result.

   a. 20° b. 45°

5. A *right angle* measures exactly 90°. Drag to make ∠**BAG** and ∠**SAK** right angles. Sketch your results below.

   a. m∠**BAG** = 90° __________ b. m∠**SAK** = 90° __________

6. Open the sketch Angles 2 (Mac) or Angles 2.gsp (Windows).
7. To measure an angle in Sketchpad, do the following steps:
   - Hold down the Shift key.
   - Click on three points: a point on either side, the vertex, and the point on the other side, in clockwise or counterclockwise order. Be sure the vertex is the middle point selected.
   - Go to the **Measure** menu and choose **Angle**. You should see the measure of the angle appear on the screen.

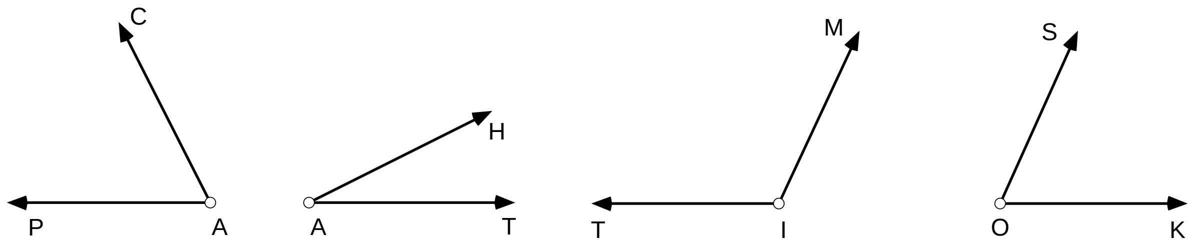
   What kind of angle is ∠**RAN** (*acute, right,* or *obtuse*)? __________

   What is the measure of ∠**RAN**? __________

   You should have found m∠**RAN** = 70°, which means it is an acute angle.
8. For each angle listed below, do the following:
   - Write whether you think it is *acute, right,* or *obtuse.*
   - Estimate the measure of the angle.
   - Use Sketchpad to measure the angle, and record its actual measure.
   - Change your answer for the kind of angle, if necessary.

	Angle	Acute, right, or obtuse	Estimated Measure	Actual Measure
a.	∠**CAT**			
b.	∠**PAT**			
c.	∠**RAP**			
d.	∠**CAP**			
e.	∠**CAB**			
f.	∠**DAN**			
g.	∠**BAP**			
h.	∠**TAD**			
i.	∠**RAD**			
j.	∠**BAR**			

Exploring Special Pairs of Angles



angle(CAP) = 63° angle(HAT) = 27° angle(MIT) = 114° angle(SOK) = 66°

In geometry, you study several special kinds of pairs of angles. Use Sketchpad to explore some of these pairs.

1. Open the sketch Pairs 1 (Mac) or Pairs1.gsp (Windows).
2. Notice ∠**CAP** and ∠**HAT**. They are *complementary angles.* As you drag point **X**, the angles will remain complementary. What do you notice about the sum of their measures? ______________________________

 ____
3. Use your observation from step 2 to define complementary angles.
4. Open the sketch Pairs 2 (Mac) or Pairs2.gsp (Windows).
5. Notice ∠**MIT** and ∠**SOK**. They are *supplementary angles.* As you drag point **G**, the angles will remain supplementary. What do you notice about the sum of their measures? ______________________________

 ____
6. Use your observation from step 5 to define supplementary angles.

 ____
7. Open the sketch Pairs 3 (Mac) or Pairs3.gsp (Windows).
8. Notice that four angles are formed by a pair of intersecting lines. In this sketch, line **BK** intersects line **GS** to form the following angles: ∠**BAG**, ∠**SAK**, ∠**BAS**, and ∠**GAK**.
9. ∠**SAK** and ∠**BAG** are a pair of *vertical angles.* How do the measures of the angles compare? ______________________________

 ____

10. ∠**BAS** and ∠**GAK** are also vertical angles. How do their measures compare?

____

11. Drag points **S** and **K**. As you drag, the pairs of angles remain vertical angles. What conclusion can you draw about the measures of any pair of vertical angles? ______________________________________

____

12. What kind of angles are ∠**BAS** and ∠**SAK**? ____________________

What kind of angles are ∠**BAG** and ∠**GAK**? ____________________

13. From what you now know about vertical and supplementary angles, answer the following question: When two lines intersect, four angles are formed. How many angle measures would you need to know before you could name the measures of all the angles? __________

Draw an example and write a short explanation to show how to do this.

____

____

____

____

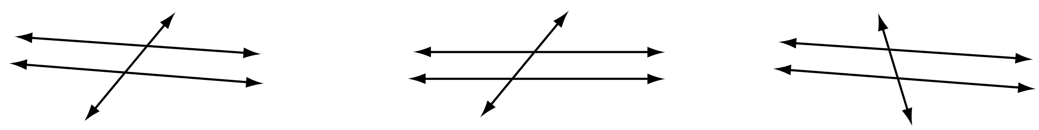
____

____

____

____

Exploring Angles Formed by Parallel Lines and a Transversal



When parallel lines are intersected by a third line, called a *transversal,* there are certain relationships among the angles formed. Use Sketchpad to explore these relationships.

1. Open the sketch Parallel (Mac) or Parallel.gsp (Windows).
2. Find ∠**HEX** and ∠**EOY**. They are a pair of *corresponding angles.* How do their measures compare? ______________________________
3. For each angle given below, list its corresponding angle and tell how the measures compare.
   a. ∠**HEM** ______________________________
   b. ∠**MEO** ______________________________
   c. ∠**XEO** ______________________________
4. Drag point **N** slowly and notice the measures of each pair of corresponding angles. How do their measures always compare? ______________________________

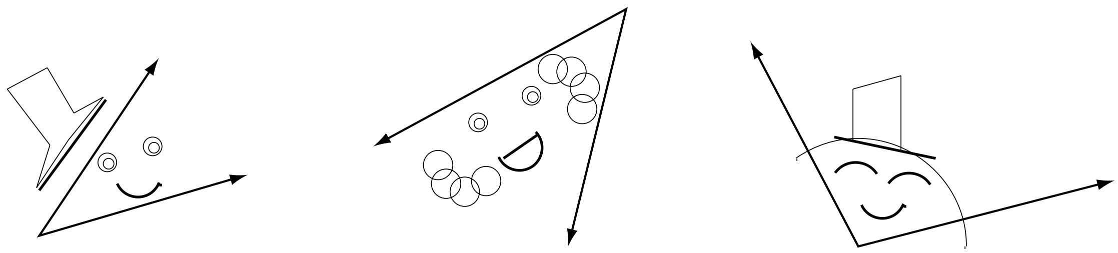
   ____

   Complete the following statement: If two parallel lines are cut by a transversal, then the corresponding angles are ______________________________
5. Find ∠**MEO** and ∠**EOY**. They are a pair of *alternate interior angles.* How do their measures compare? ______________________________
6. Name the other pair of alternate interior angles in the sketch: ____________ and ____________. How do their measures compare? ______________________________

   ____

7. Drag point **N** slowly and notice the measures of both pairs of alternate interior angles. How do their measures compare? ______________
   Complete the following statement: If two parallel lines are cut by a transversal, then the alternate interior angles are ______________
8. Find ∠**HEM** and ∠**YON.** They are a pair of alternate exterior angles. How do their measures compare? ______________
9. Name the other pair of alternate exterior angles in the sketch: ____________ and ____________. How do their measures compare?
   ____
10. Drag point **N** slowly and notice the measures of both pairs of alternate exterior angles. How do their measures always compare? ______________
    ____
    Complete the following statement: If two parallel lines are cut by a transversal, then the alternate exterior angles are ______________
    ______________.

Interesting Angles



Kyle Ashley Clint

1. Open a new sketch.

   Use Sketchpad to construct the following:

   a. a pair of angles that intersect at one point
   b. a pair of angles that intersect at two points
   c. a pair of angles that intersect at three points
   d. a pair of angles that intersect at four points
   e. a pair of angles that intersect at an infinite number of points
   f. a pair of angles that intersect at any other number of points

2. If you used the Segment tool to construct your angles and you wish to make arrows at the ends of the segments, you may do the following:

   a. Set the **Script Tool** folder to Utility Scripts (Mac) or Utilities (Windows). (See the Points, Lines, and Angles Quick Reference Guide for how to do this.)
   b. Press and hold down the mouse button on the Script tool . Choose **Open Arrow** from the menu.
   c. Start at the vertex of the angle and drag to the point where you wish to construct the arrowhead.

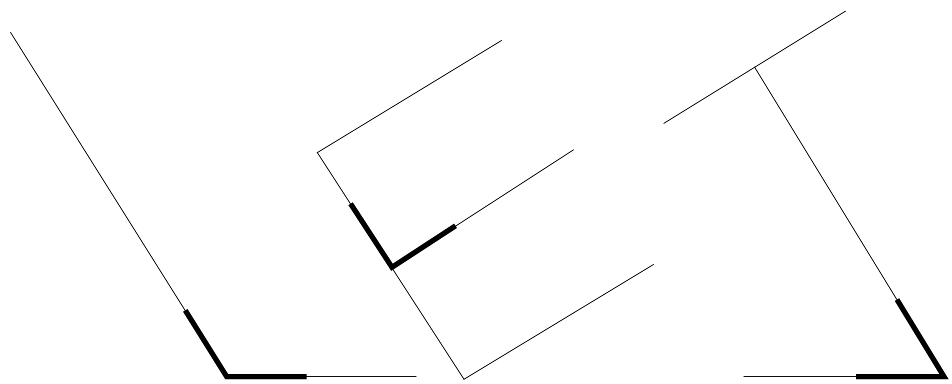
3. Put your name in the sketch and print out a copy to hand in.

Challenge: In how many points can three angles intersect? Use Sketchpad to investigate.

Name Angles

1. Open a new sketch.
2. Use the Segment tool to write your initials in the sketch.
3. Display the measures of three angles in your sketch. Try to show one acute, one obtuse, and one right angle, if possible.

Example:



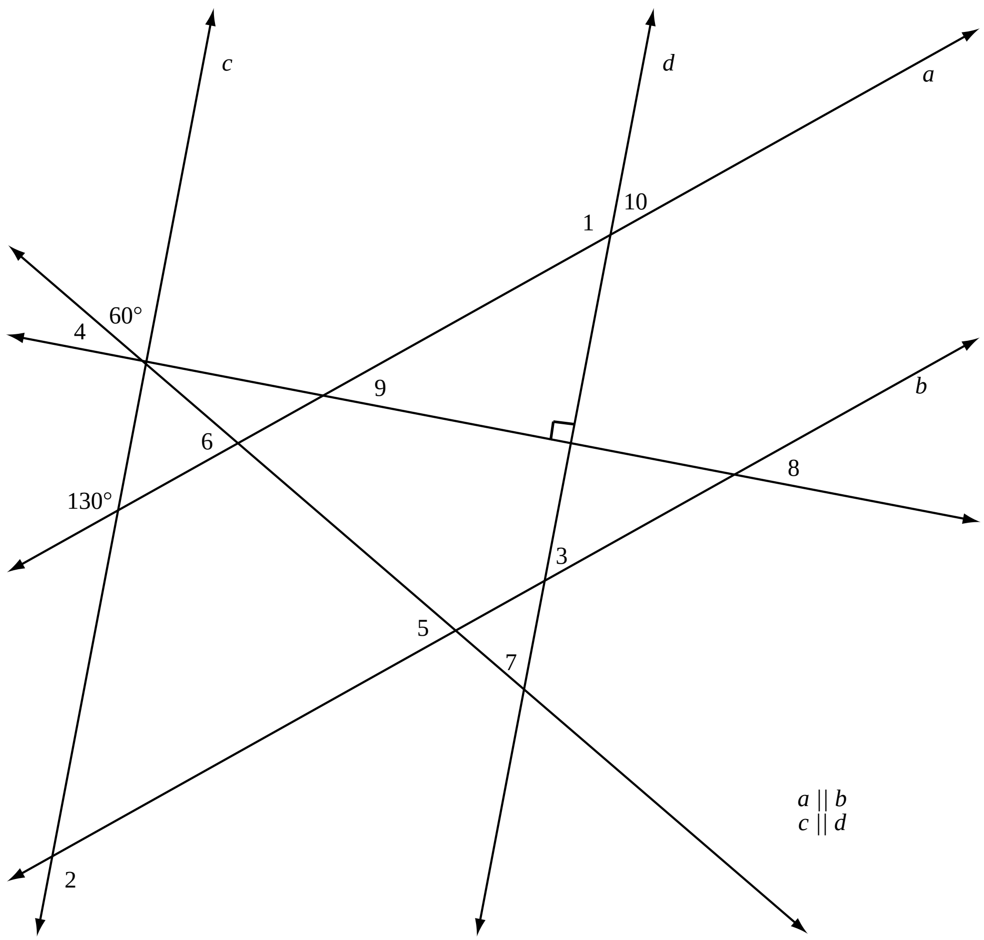
Lathem Emily Jenkins

L's angle measure = 122° **E**'s angle measure = 90° **J**'s angle measure = 58°

4. Put your name in the sketch and print out a copy to hand in.

Angle Puzzle

1. Using only the information you are given in the drawing below, work with the students in your group to find the measure of each numbered angle.
2. Use what you know about the measures of special pairs of angles and angles formed by parallel lines and a transversal to help you. Also, remember that the sum of the angles of any triangle is 180°.



m∠1 = ________ m∠2 = ________ m∠3 = ________ m∠4 = ________

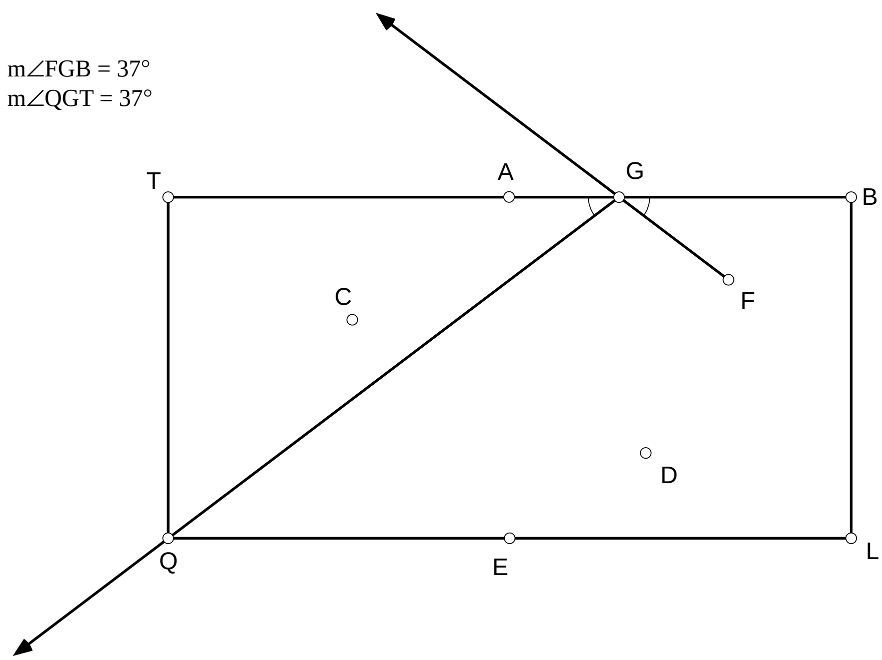
m∠5 = ________ m∠6 = ________ m∠7 = ________ m∠8 = ________

m∠9 = ________ m∠10 = ________

Challenge: Use Sketchpad to design your own Angle Puzzle. You must be careful to construct parallel lines that will stay parallel. Also, try only a few angles at first! Have another student try your first draft, and don't be surprised if you have to revise.

Sharp Shooter

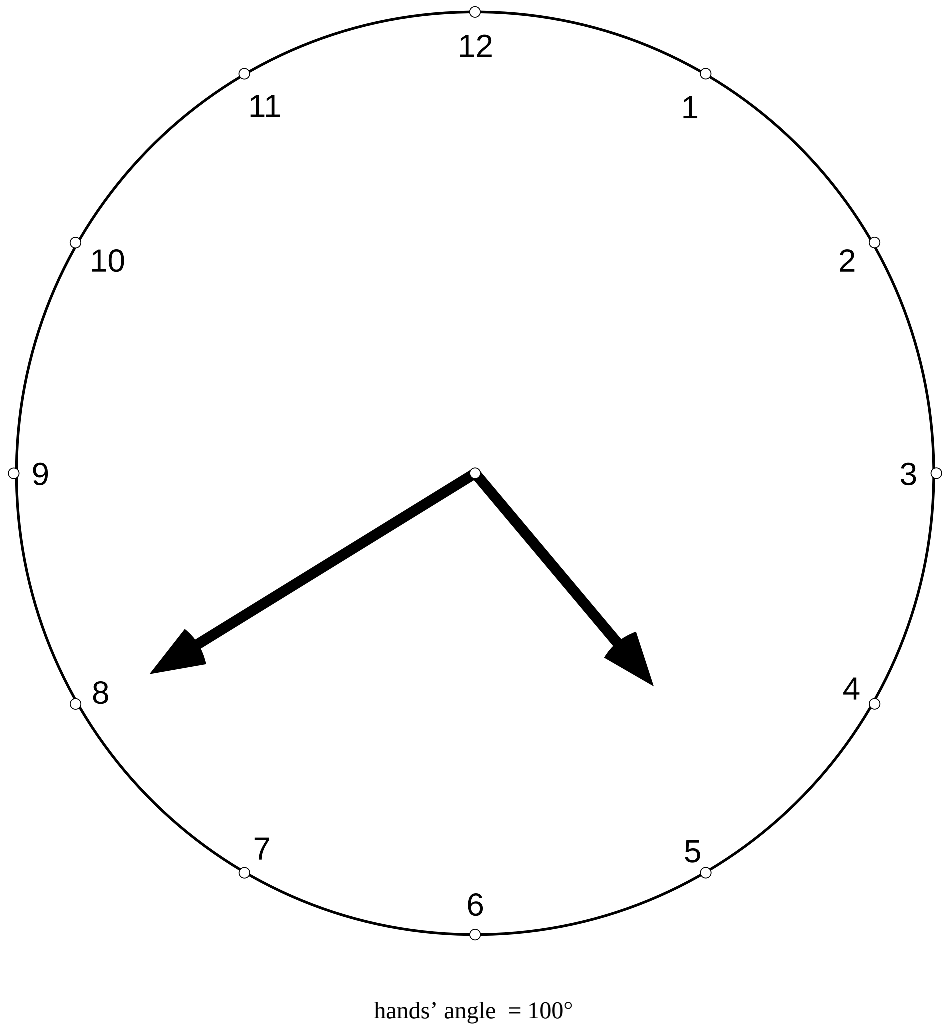
The figure below is a scale drawing of a pool table, with points **C**, **D**, and **F** representing balls. Points **T**, **A**, **B**, **L**, **E**, and **Q** represent pockets. Suppose you want the ball to bounce off one of the sides of the pool table and roll into one of the pockets. Where on the table would you aim for the ball to strike? Remember that the path of a ball must make equal angles with a side of the pool table to model or simulate a real pool ball, as shown in the figure.



1. Open the sketch Pool (Mac) or Pool.gsp (Windows).
2. Use the Ray tool to construct a path for one of the balls. The ball must be the endpoint of your ray, and the other control point must be constructed on the side at which you decide to aim. Construct a second ray with the intersection point of the first ray and the side as its endpoint and the other control point the pocket for which you are aiming. Measure the angles made by the path of the ball and the side of the pool table. Drag the point on the side to adjust the measures of the angles until they are congruent.
3. Print out a copy with your results.
4. Repeat this for the other two balls.
5. Go to the **Edit** menu and choose **Undo** until you see the original pool table on the screen. This time, try to find the path of a ball that bounces off two sides and then enters a pocket.

Creating a Sketchpad Clock

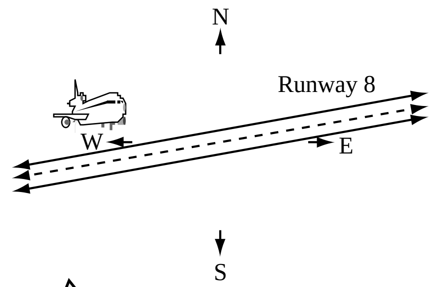
1. Open a new sketch and create an accurate model of a clock.
2. Display correct measures for any time of day.



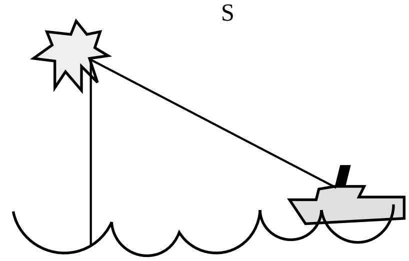
Challenge: Animate the hands of your clock so that they move just like the hands of a real clock. Make sure the hands rotate clockwise.

More Ideas for Points, Lines, and Angles Projects

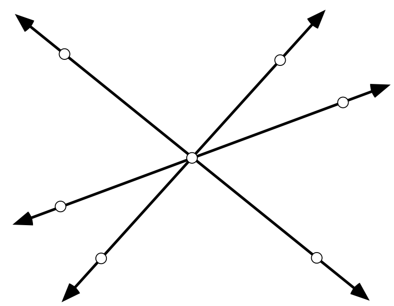
1. Investigate how runways are numbered at airports. Use Sketchpad to illustrate how angles determine the numbers and what they mean to pilots.



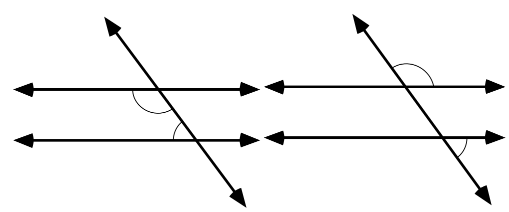
2. Investigate how angles are used in celestial navigation. Use Sketchpad to create a sketch that illustrates this means of determining your position.



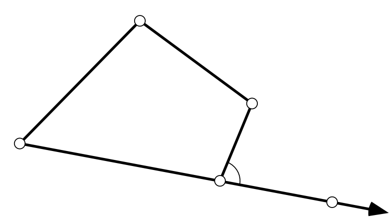
3. Use Sketchpad to construct three lines intersecting at a single point to form six angles. Construct a point on each ray that doesn't already have a point on it. Notice special pairs of angles formed. Experiment to discover how many angle measures you would need to know in order to find the rest.



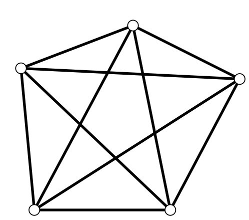
4. Investigate the relationship between the measures of pairs of consecutive interior angles formed by a pair of parallel lines and a transversal. Also, investigate the relationship between the measures of pairs of consecutive exterior angles formed by a pair of parallel lines and a transversal.

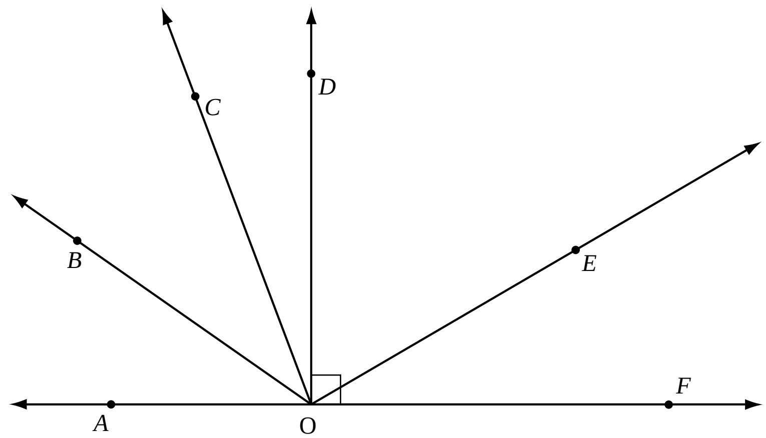


5. Explore the relationship between the measures of an exterior angle of a triangle and the two remote (nonadjacent) interior angles. Then investigate the relationship between the measures of an exterior angle of a quadrilateral and the three nonadjacent interior angles.



6. Explore with diagonals in polygons to find a pattern for the maximum number of intersection points of the diagonals of a convex polygon.



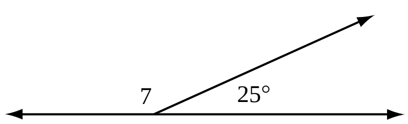


Tell whether each angle listed below is acute, right, or obtuse.

1. ∠COD ______________
2. ∠BOE ______________
3. ∠DOA ______________
4. ∠FOE ______________
5. ∠EOA ______________
6. ∠AOC ______________

For each pair of angles in questions 7–10, (a) identify the pair as complementary, supplementary, or vertical and (b) give the measure of the numbered angle.

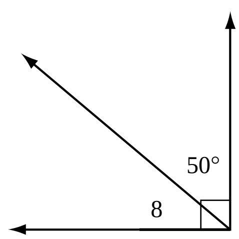
7.



a. ______________

b. ______________

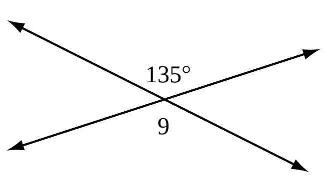
8.



a. ______________

b. ______________

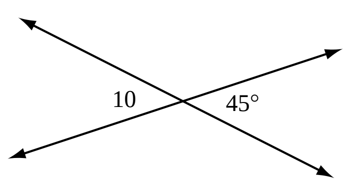
9.



a. ______________

b. ______________

10.

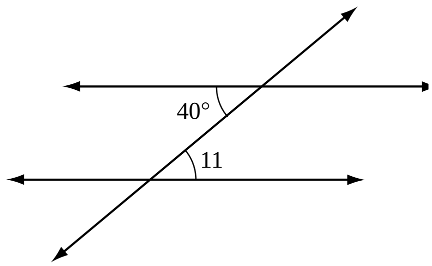


a. ______________

b. ______________

For each pair of angles in questions 11–14, (a) classify the pair as corresponding angles, alternate interior angles, or alternate exterior angles and (b) give the measure of the numbered angle. For these questions, the two horizontal lines are parallel.

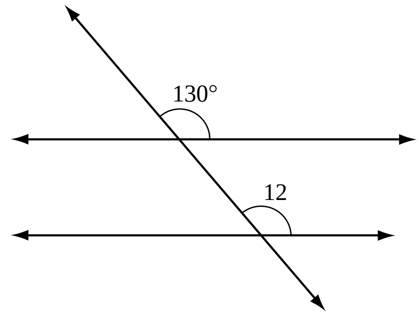
11.



a. ______________

b. ______________

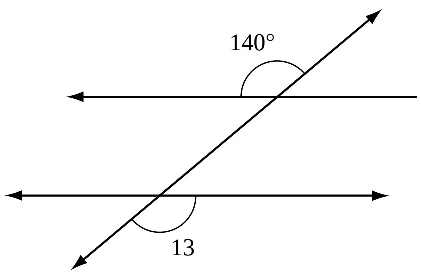
12.



a. ______________

b. ______________

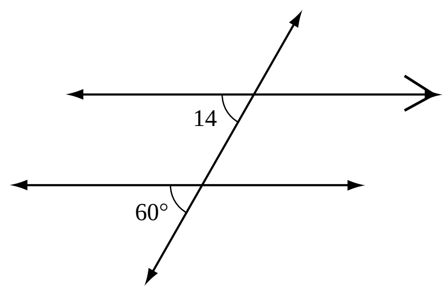
13.



a. ______________

b. ______________

14.



a. ______________

b. ______________

Write *sometimes, always,* or *never* for each statement in questions 15–20.

15. ________________ A pair of angles are vertical and acute.

16. ________________ Alternate interior angles formed by parallel lines and a transversal are complementary.

17. ________________ A pair of angles are vertical and supplementary.

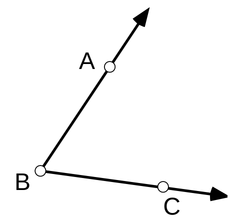
18. ________________ Corresponding angles formed by parallel lines and a transversal are congruent.

19. ________________ Alternate exterior angles formed by parallel lines and a transversal are vertical angles.

20. ________________ One of a pair of supplementary angles is acute and one is obtuse.

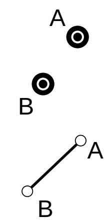
To measure an angle (use the Selection Arrow tool)

1. Hold down the Shift key and click on the three points that define the angle. (Make sure you click on the vertex second. For **B**, click on **A**, **B**, and then **C**, or click on **C**, **B**, and then **A**.)
2. Go to the **Measure** menu and choose **Angle**.



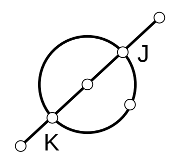
To construct a segment between two existing points
(use the Selection Arrow tool)

1. Hold down the Shift key and click on the two points.
2. Go to the **Construct** menu and choose **Segment**. (If the **Construct** menu shows **Ray** or **Line** instead of **Segment**, change the Ray or Line tool to the Segment tool on the toolbox.)



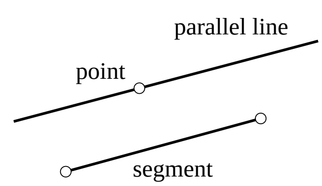
To construct the point(s) of intersection for two objects
(use the Selection Arrow tool)

1. Hold down the Shift key and click on the two objects.
2. Go to the **Construct** menu and choose **Point At Intersection**.



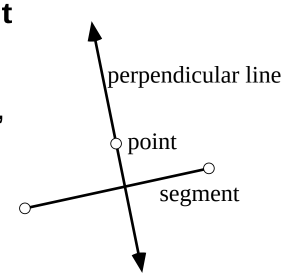
To construct a line parallel to a given line, ray, or segment through a given point (use the Selection Arrow tool)

1. Hold down the Shift key and click on the point and the given line, ray, or segment.
2. Go to the **Construct** menu and choose **Parallel Line**.

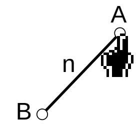


To construct a line perpendicular to a given line, ray, or segment through a given point (use the Selection Arrow tool)

1. Hold down the Shift key and click on the point and the given line, ray, or segment.
2. Go to the **Construct** menu and choose **Perpendicular Line**.

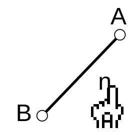


To display or hide a label (use the Text tool)



1. Position the Text tool over the object and click once. For a segment, click on the segment itself, not on the endpoints.
2. To hide the label, click on the object again (not on the label).

To move a label (use the Text tool)



1. Click on the label and drag it to any location close to the object.

To change a label name (use the Text tool)

1. Double click on the label.
2. When the **Relabel** dialog box appears, rename the object.

Relabel Dialog Box (Windows)

To write your name(s) on a sketch and then print (use the Text tool)

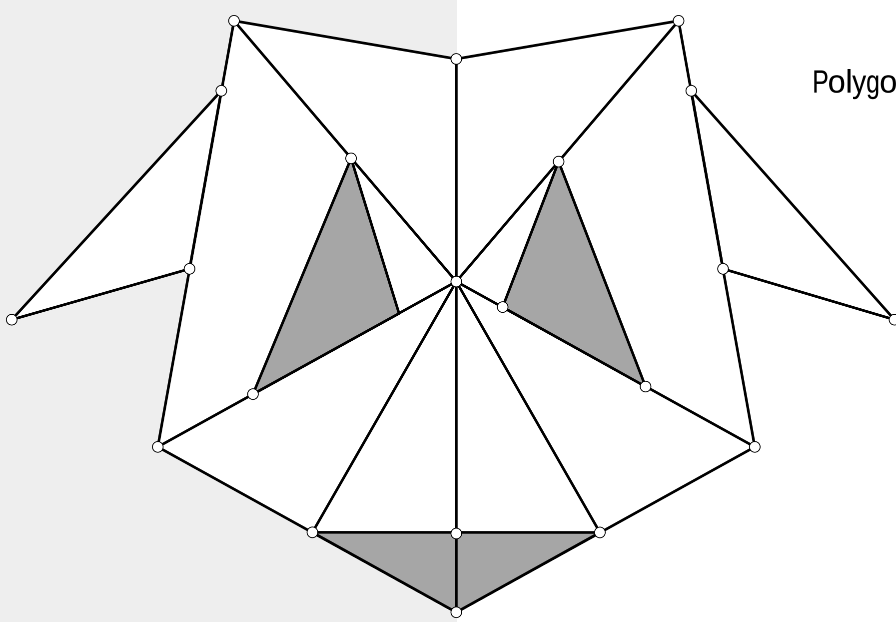
1. Drag in a blank area of your sketch to make a text box.
2. Type your name(s) in this box.
3. Go to the **File** menu and choose **Print.**

To set the Utility Scripts **folder as the** Script Tool **folder**

1. Go to the **Display** menu and choose **Preferences**.
2. Click on **More**.
3. In the **Script Tool Folder** box, click on **Set**. (You may have to click on **Clear** first.)
4. Find and open the Utility Scripts folder (Mac) or Utilities directory (Windows). This folder comes with The Geometer's Sketchpad Sample Scripts.
5. Click on **Select "Utility Scripts"**.
6. Click on **Continue** to dismiss the **Advanced Preferences** dialog box.
7. Click on **OK** to dismiss the **Preferences** dialog box. You should see the Script tool added to the bottom of the toolbox.

Unit 3

TRIANGLES



Commentary

The skill levels required by the activities in the Triangles unit probably reflect the widest range of any in this book. The activities and projects range from **Angles** (which is appropriate for students who have little or no experience with triangles) to **Pythagoras Plus** (which is a discovery activity for many mathematics teachers!).

Throughout this unit, you need to emphasize the effects of rounding numbers in general and when using Sketchpad. For example, while using the sketch Pythagoras 2 (Mac) or Pythag2.gsp (Windows), where side lengths are displayed to the nearest tenth and angle measures are displayed to the nearest unit, the sketch may display **a** = 3.0, **b** = 4.0, and **c** = 5.0 and yet may display ∠**NOW** as 91°. Preferences are set and saved in the sketches provided with this unit to help minimize the effects of rounding, but students must learn to deal with this reality. As you probably experience in your own classroom, we find that some middle-school math students need several reminders of this particular feature!

When your students do **Altitudes,** encourage them to drag angles **H**, **J**, and **K** to make each of them obtuse or right before formulating their conjectures. For **Area,** you may need to remind some students that Sketchpad will display the area of a polygon if they select the polygon interior and choose **Area** from the **Measure** menu. It may be appropriate to show the activity **Angles** as a teacher demonstration whether or not you choose to include the section dealing with exterior angles. The sum of angle measures in a triangle is also covered in **Exploring Properties of Triangles,** so you will want to choose which activity is more appropriate for your students.

Students for whom **Exploring Properties of Triangles** is appropriate may need a detailed teacher demonstration, since most of them have had little experience with either Sketchpad or formal geometry. Also, this activity is likely to be one of the first encounters with Sketchpad for many students. The portion of the activity that deals with the sum of the angle measures can be a class demonstration for some more experienced groups. For others, it will be primarily a reminder. However, for some beginner groups, the activity will be entirely new; you may want to add a follow-up demonstration by tearing one or more paper triangles and arranging the three angles to form a straight angle to help establish the concept. At the other extreme, the portion of this activity that deals with triangle inequalities, which seem quite obvious to most teachers, seems to surprise even many students who have had a good bit of classroom experience with geometry before encountering Sketchpad. For students at all skill levels, the discovery method Sketchpad affords makes it much easier to remember to test the sum of each pair of sides against the length of the third side to determine whether the triangle is possible. With a textbook or even a chalkboard demonstration, our students in the past nodded when the idea was presented but repeatedly forgot it in follow-up activities or written assessments. This portion of the activity seems a must for groups at all levels.

Before trying **Exploring Types of Triangles,** students need to understand clearly the classification of triangles by sides and by angles; only then can they discover the relationships among the characteristics of triangles as defined by these classifications through practice with the sketches. It was gratifying to hear the discussions between and among pairs as they worked: "Look, you can tell that every equilateral triangle is also isosceles because any two sides you choose are already the same length" "Of course a right triangle has two acute angles—you have only 90° left for both of them" and "OK, show me a scalene obtuse triangle, Mr. Smartie!" We have yet to have a group of middle-schoolers for whom this activity is not beneficial.

Exploring the Pythagorean Theorem is designed to introduce students to the theorem.

Again, the teacher demonstration plays an important role for most students, because the activity incorporates building a table with Sketchpad, a skill most middle-school students either have not encountered or do not use often enough to recall without a reminder. With students who have had some exposure to the theorem, it was surprising to us how many of our students assumed the relationship would also exist among the side lengths in nonright triangles. Of course, if you have time constraints, you could split this activity into two parts, but we have encountered misunderstanding among some students when we have rushed or omitted the second half. The informal exposure to converses in the second part of this activity is truly enriching for many students.

For **Triangle Search** to be successful, we have found it helpful for kids to work in pairs and then compare results with at least one other pair. The benefits of cooperation have been very obvious to our students during this activity, since different results were invariably obtained on the first attempt. In fact, this activity helped win over to this approach some students formerly reluctant to work with others. Also, you need to emphasize to your students that they must avoid dragging the figures and to remind them that they can use the **Edit Undo** feature if they accidentally alter the sketch. After completing the activity, our students enjoy dragging the design into other shapes. Alternatively, you may want to design a more constrained figure for some groups of students.

The activity **The Real Real Number Line** is more challenging than most others in this book. For many middle-school classes, it may be appropriate to use it only as a teacher demonstration. For others, we have gone through almost the entire activity with a student sitting at the computer while the teacher verbally led students through the activity. In this case, the focus was on students' gaining Sketchpad skills and on reinforcing the location of commonly encountered irrational numbers when they did their own follow-up constructions. Pairs work nicely here. With more advanced groups of students, of course, a shorter, less detailed demonstration is needed. Regardless of their level, every group of middle-schoolers requires some discussion about radicals and irrational numbers prior to carrying out the activity. The stage for this activity is set with questions such as "What are irrational numbers?" "Where on the number line is $\sqrt{9}$?" and "Between what two whole numbers will you find $\sqrt{18}$?" Regardless of the format in which it is used, we have been pleased with the impact of this activity. It is the first time the Real Number line truly becomes real for students. Students repeatedly comment, "Now the number line in my head is fuller," or "Constructing my own Sketchpad number line helped me remember what $\sqrt{2}$ is." If your paper supply is generous, it is helpful to have students print an extra copy of their number line showing all hidden features. We often ask students to save a copy of their sketches on a disk so we can check them for verification or construction errors.

Slice It, Dice It can be extended nicely for algebra students. If the drawings are done on a grid, questions and observations about slope are appropriate and interesting.

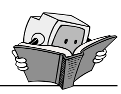
Kids really get into **PT Problems** as a follow-up to **Exploring the Pythagorean Theorem.** Having students present their problems in class for others to solve is a motivating, effective method of alternative assessment. Again, pairs work well here.

Don't overlook **Pythagoras Plus.** Students of all ages (including most adults) feel like "real mathematicians" as they discover the extension of the Pythagorean Theorem on their own. It is a wonderful attribute of Sketchpad that a student who lacks the skill to find the area of regular polygons beyond squares can have the same feeling of accomplishment as others with more experience, since Sketchpad will calculate and display the area for the student. For some kids, scripts for construction of the regular polygons may be needed; for others, doing their own constructions and/or scripts is the highlight of the activity.

Before students attempt the project **A Look Inside,** you need to demonstrate the Sketchpad skill of constructing a midpoint of a side and then a new segment for each half. This project is technically more difficult than most projects in this book. It fits nicely into a fractals unit.

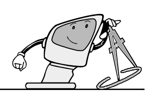
If your students explore tessellations with triangles (see number 3 in **More Ideas for Triangles Projects**), we have found that exploring with equilateral triangles first is the most successful route for most middle-school students, because it is technically so much easier. Most students can use the Scripting tool to tessellate equilateral triangles. Of course, you can approach this topic by tessellating first with a scalene triangle. Then the students can drag a vertex and watch the triangles change from scalene to isosceles or equilateral if their Sketchpad proficiency is sufficient.

Prerequisite Mathematical Terms and Concepts



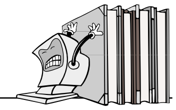
- *vertex, ray, acute angle, right angle, obtuse angle*
- names and rules for types of triangles classified by sides (scalene, isosceles, equilateral) and by angles (acute, obtuse, right)

Recommended Sketchpad Proficiency



- Basic knowledge of the freehand tools.
- Use of the Polygons Quick Reference Guide at the end of this unit is suggested.
- Doing the Points, Lines, and Angles unit prior to this one would be beneficial; otherwise, you might want to use specific sketches from that unit as student questions arise.

Essential Vocabulary



Acute triangle—a triangle with three acute angles

Equilateral triangle—a triangle with all sides congruent

Exterior angle—any angle formed outside a polygon by one side of a polygon and the extension of an adjacent side

Hypotenuse—side opposite the right angle in a right triangle

Interior angle—any angle formed within a polygon by two adjacent sides

Isosceles triangle—a triangle with at least two congruent sides

Leg—one of the sides forming the right angle in a right triangle

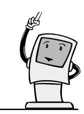
Obtuse triangle—a triangle with one obtuse angle

Right triangle—a triangle with one right angle

Scalene triangle—a triangle with no congruent sides

Triangle—a polygon with three sides

Instructions for Teacher Demonstrations



In each unit, we have included sketches and guidelines for teacher demonstrations designed as introductions by the teacher in whole-class presentations before students attempt the various activities independently. They include the introduction of relevant mathematical vocabulary and concepts as well as appropriate Sketchpad skills. Often there are several activities that you will feel comfortable assigning to your students with very little guidance beforehand. In this unit, for example, the activities **Altitudes** and **Angles** work fine as exploratory introductions to those topics and require very little technical expertise. For **Area,** on the other hand, students should know the formula for finding the area of a triangle. In addition, you might want to demonstrate how to use Sketchpad to display area before assigning the activity. One or more activities in each unit are designed to maximize guided discovery by the students. For these activities, a careful teacher demonstration will focus the students' attention on targeted mathematical concepts and prevent distractions due to lack of Sketchpad proficiency. In this unit, the activities **Exploring Properties of Triangles, Exploring Types of Triangles,** and **Exploring the Pythagorean Theorem** are designed to follow such a demonstration. Instructions are included later in these notes, along with answers for the guided discovery activities, for your convenience. Of course, you may want to use portions or adaptations of the demonstrations before other activities; certainly, you will want to preview activities to be sure they are appropriate for the mathematical and Sketchpad skill levels of your students. The teacher demonstrations will help ensure success with most of the activities in this unit, since the topics related to triangles come early in most middle-school geometry curricula, and needed Sketchpad skills, which are introduced in each portion of the demonstration, may be new to many of your students. Each new Sketchpad skill may require several repetitions. We often show several examples and then, at the close of each teacher demonstration, have a student sit at the computer while another student gives verbal instructions.

You may find the Polygons Quick Reference Guide at the end of this unit helpful in preparing for the triangles demonstrations.

Exploring Properties of Triangles

1. Demonstrate the following Sketchpad skills using Triangles Demo 1 (Mac) or Tridemo1.gsp (Windows):
   - Drag a vertex using the Selection Arrow tool.
   - Move labels using the Text tool.
   - Move captions or measures using the Selection Arrow tool.
   - Find the sum of angles using **Calculate** in the **Measure** menu (you may want to show students how to double click on a measure to activate the calculator).
2. Demonstrate the following Sketchpad skills using Triangles Demo 2 (Mac) or Tridemo2.gsp (Windows):
   - Manipulate segments to given lengths.
   - Make endpoints of segments meet (or not) to form vertices of a triangle.

Exploring Types of Triangles

1. Discuss classifications of triangles by sides and by angles. Open Triangles Demo 3 (Mac) or Tridemo3.gsp (Windows) and drag to illustrate each of the following:
   - Scalene triangle
   - Isosceles triangle
   - Equilateral triangle
   - Acute triangle
   - Obtuse triangle
   - Right triangle
2. Explain that the triangle in each sketch used by students in **Exploring Types of Triangles** (with the exception of the scalene triangle) will remain that kind of triangle when dragged; in other words, in the isosceles triangle sketch, the triangle will always remain isosceles, and in the equilateral triangle sketch, the triangle will always remain equilateral. For the scalene triangle sketch, emphasize to students that they should observe the measures of the sides to keep the triangle scalene as they drag to answer the questions.

Exploring the Pythagorean Theorem

1. Review angles, if needed. Open a new sketch. Use the Segment tool to construct an angle. Drag to show *acute, right,* and *obtuse* angles.

2. Review right triangles, including the terms *hypotenuse* and *legs.* Open Triangles Demo 4 (Mac) or Tridemo4.gsp (Windows) and drag to illustrate each of the following Sketchpad skills:
   - Build a table using **Tabulate** in the **Measure** menu.
   - Manipulate segments to given lengths.

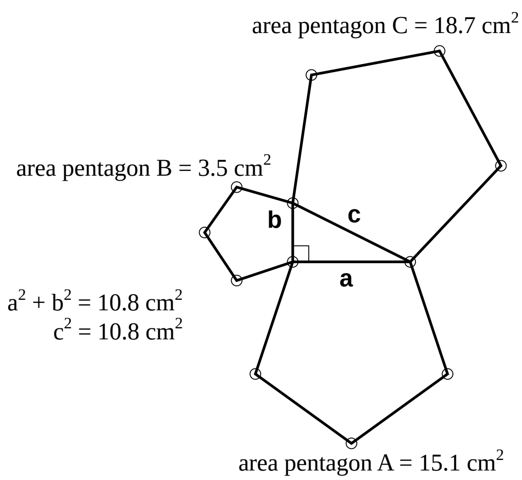
Writing Prompts

Ask students to choose a topic below and to write a short paper, including sketches created with Sketchpad.

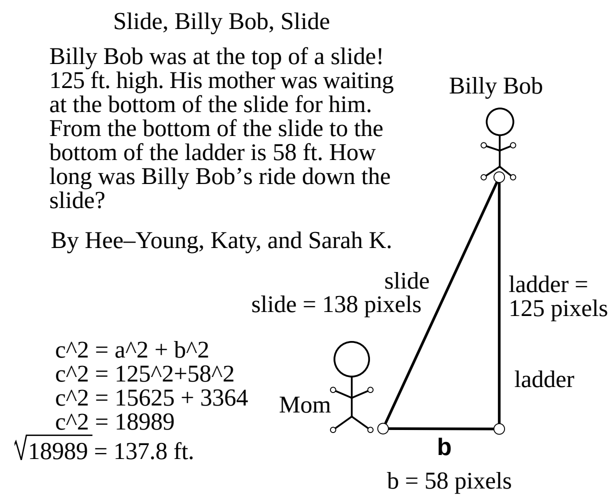
- All I Know About Triangles
- Triangles Everywhere
- Pythagoras Lives
- Triangles Are Confusing!?

Examples of Student Work

Triangles Project: Pythagoras Plus



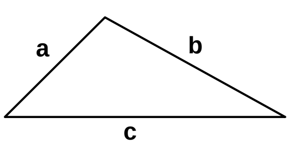
Triangles Project: PT Problems



Answers for Exploring Properties of Triangles Activity

Be sure you discuss with students the effects of rounding in this activity.

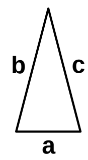
6. The sum of the angle measures in any triangle is 180°.
7. a. The angle measures are also equal.
   b. The angle measures are not equal.
   c. The lengths of the two sides are also equal.
   d. The lengths of those two sides are not equal.
8. Answers will vary. One acceptable response: The longest side in any triangle is opposite the largest angle, and the shortest side in any triangle is opposite the smallest angle.
11. a.



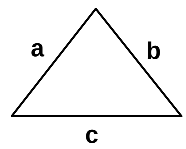
b. impossible

c. impossible

d.



e.



f. impossible

12. Answers will vary. One acceptable response: The sum of the lengths of two sides was less than the length of the third side.
13. Answers will vary. One acceptable response: The sum of the lengths of any two sides of a triangle is always greater than the length of the third side.

Answers for Exploring Types of Triangles Activity

Be sure you discuss with students the effects of rounding in this activity.

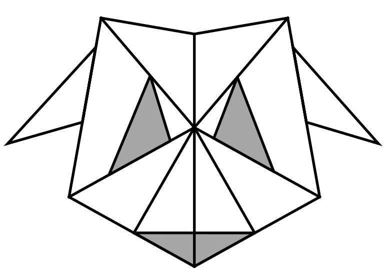
2. a. yes b. yes c. yes
4. a. yes b. yes c. yes
6. a. yes b. no c. no
7. a. sometimes b. sometimes c. never
   d. never e. never f. sometimes
   g. always h. always

Answers for Exploring the Pythagorean Theorem Activity

Be sure you discuss with students the effects of rounding in this activity.

4. For each right triangle, the sum of $\mathbf{a}^2$ and $\mathbf{b}^2$ is equal to $\mathbf{c}^2$.
6. (Wording may vary.) In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.
9. a. no, no, 90° b. no, no, 90° c. no, no, 90°
   d. no, no, 90° e. no, no, 90° f. no, no, 90°
10. a. yes b. no c. yes, yes, no
11. (Wording may vary.) A triangle will be a right triangle only when the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Solution for Triangle Search Activity



acute triangles <u>11</u>

obtuse triangles <u>9</u>

right triangles <u>4</u>

number of different sizes of triangle <u>13</u>

scalene triangles <u>12</u>

isosceles triangles <u>12</u> (including equilateral triangles)

equilateral triangles <u>2</u>

total number of triangles <u>24</u>

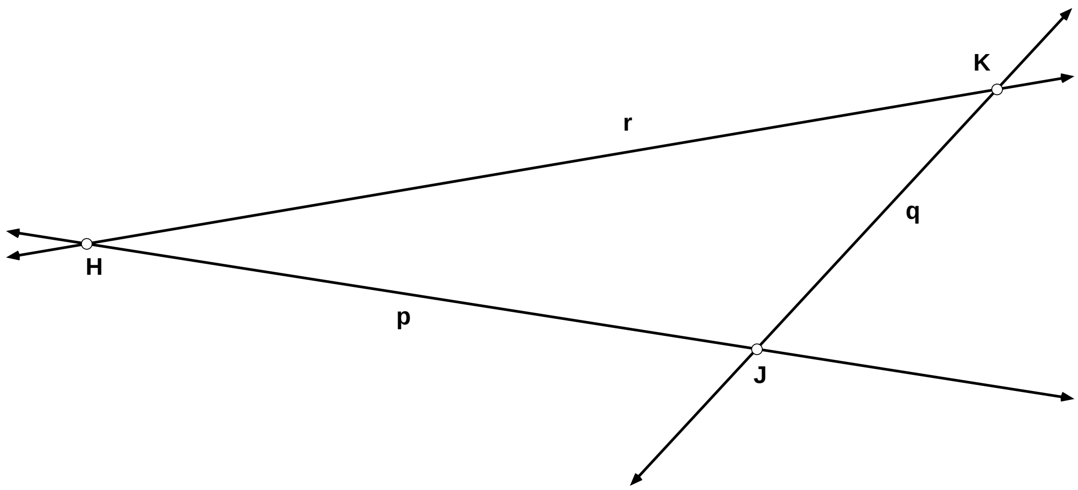
Answers for Triangles Wrap-Up

1. yes	2. no	3. yes
4. **b**, **c**, **a**	5. **c**, **a**, **b**	6. **a**, **b**, **c**
7. **c**, **b**, **b**	8. **c**, **b**, **a**	9. **a**, **b**, **c**

10–15. Drawings will vary. Triangles described in questions 13 and 14 are impossible.

Altitudes

1. Open the sketch Tri Altitudes (Mac) or Tri_alt.gsp (Windows).



2. Construct a perpendicular line from any vertex of the triangle to the opposite side. Change its color or make it dashed. Construct the point of intersection of the line and the side of the triangle. The segment from the vertex to this point of intersection is called an *altitude* of the triangle.
3. Drag any of the points **H**, **J**, or **K** to make an acute triangle. Describe the placement of the altitude. Write your conjecture. ____

   ____

   ____
4. Drag any of the points **H**, **J**, or **K** to make a right triangle. Describe the placement of the altitude. Does it matter which angle is the right angle? Write your conjecture.

   ____

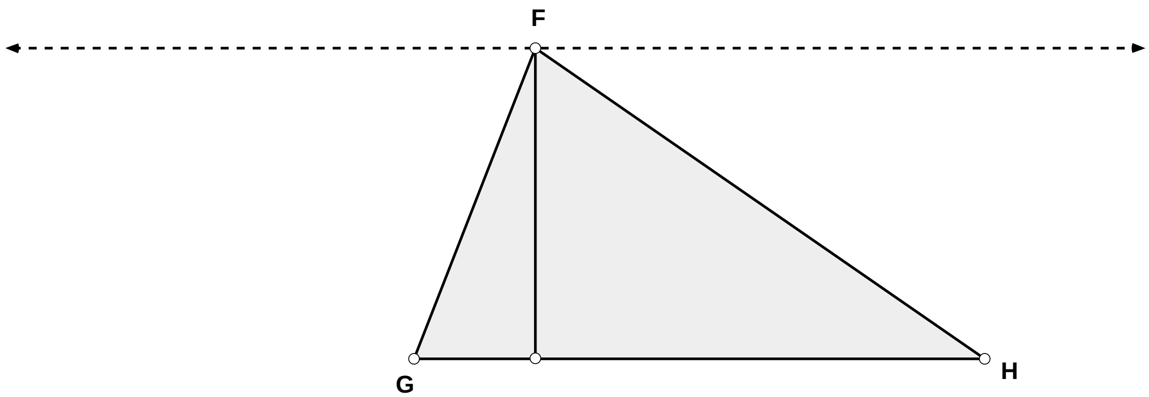
   ____
5. Drag any of the points **H**, **J**, or **K** to make an obtuse triangle. Describe the placement of the altitude. Does it matter which angle is the obtuse angle? Write your conjecture. ____

   ____

   ____

Area

1. Open the sketch Tri Area (Mac) or Tri_area.gsp (Windows).



2. Use Sketchpad to determine the area of the triangle in two different ways. Use side **GH** as the base.
3. Now drag vertex **F** or use the Animate button to change the shape of the triangle. Observe the effect on the area. Make conjectures and explain your reasoning.

   ____

   ____

   ____

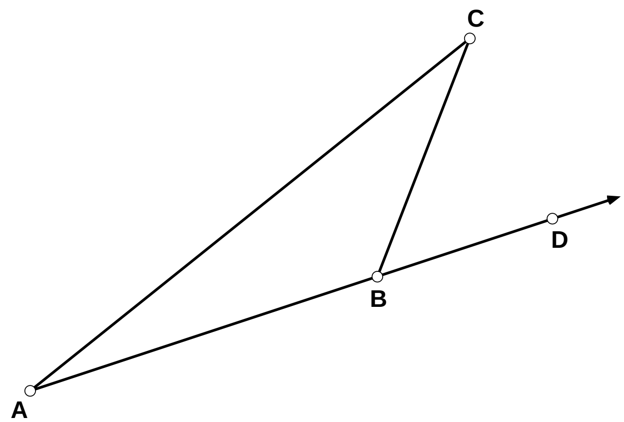
   Test your conjectures by dragging the dashed line or one of the points **G** or H. Do your conjectures still hold? Explain. ____________________

   ____

   ____

Angles

1. Open a new sketch.
2. Construct a triangle.
3. Extend one side by constructing a ray using two vertices.



4. Measure each of the interior angles.
5. Go to the **Measure** menu and choose **Calculate**. Use Sketchpad's calculator to determine the sum of the three interior angles.
6. Drag any vertex of the triangle and observe the measures of the interior angles and their sum.
7. Write any conjectures based on your exploration. ______________________________

   ____

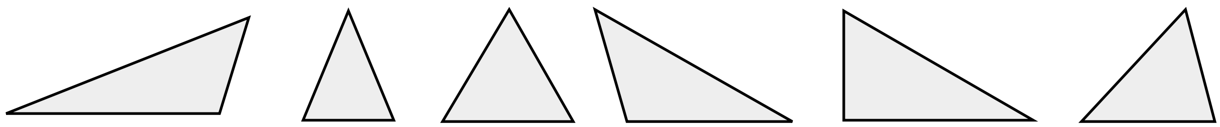
   ____

8. Click somewhere on the ray outside the triangle to construct a point. Measure the exterior angle.
9. Use Sketchpad's calculator to determine the sum of the two interior angles that are not adjacent to the exterior angle.
10. Drag any vertex of the triangle and compare the measure of the exterior angle to the sum of the two remote (nonadjacent) interior angles.
11. Write any conjectures based on your exploration. ______________________________

    ____

    ____

Exploring Properties of Triangles



1. Open the sketch Exploring Triangles 1 (Mac) or Exp_tri1.gsp (Windows).

You will use the Sketchpad calculator to find the sum of the measures of the angles in any triangle.

2. Go to the **Measure** menu and choose **Calculate**, or double click on one of the angle measures to activate the calculator.
3. Click on one of the angle measures that is showing in your sketch.
4. Click on the **+** sign and then click on another angle measure showing in your sketch.
5. Click on the **+** sign again and click on the third angle measure. Then click on **OK.**
6. Drag any vertex and observe the sum of the angle measures. What can you conclude about the sum of the measures of the angles of any triangle?

   ____

Now you will explore the measures of the angles and the lengths of the sides opposite those angles.

7. Drag any vertex to change the size and shape of your triangle. Observe the measures of the angles and the lengths of the sides. Answer the questions below.
   a. If the lengths of two sides of a triangle are equal, what do you know about the measures of the angles opposite them? ______________________

      ____

      ____

   b. If the lengths of two sides of a triangle are unequal, what do you know about the measures of the angles opposite them? ______________________

      ____

   c. If the measures of two angles of a triangle are equal, what do you know about the lengths of the sides opposite them? ______________________

      ____

d. If the measures of two angles of a triangle are unequal, what do you know about the lengths of the sides opposite them? ___________________________________

____

8. Summarize the relationship in any triangle between the measures of the angles and the lengths of the sides opposite them. _______________________________________

____

____

____

Now you will explore the relationship among the lengths of the sides of a triangle.

9. Open the sketch Exploring Triangles 2 (Mac) or Exp_tri2.gsp (Windows).

10. Drag the endpoints of the parallel segments to adjust the measures of the sides of your figure. Then swing the endpoints of the figure to see whether you can make a triangle. The endpoints must meet to form the vertices of the triangle.

11. Try to construct a triangle using the following lengths for the three sides. For each example, sketch your triangle or write *impossible* in the box.

	Side a	Side b	Side c	Triangle?
a.	2.0 cm	3.0 cm	4.0 cm	
b.	6.0 cm	1.0 cm	4.0 cm	
c.	3.5 cm	2.0 cm	6.0 cm	
d.	3.0 cm	4.0 cm	4.0 cm	
e.	5.0 cm	5.0 cm	6.0 cm	
f.	2.0 cm	7.0 cm	4.0 cm	

12. Why was it impossible to construct a triangle with some of the given lengths?

____

____

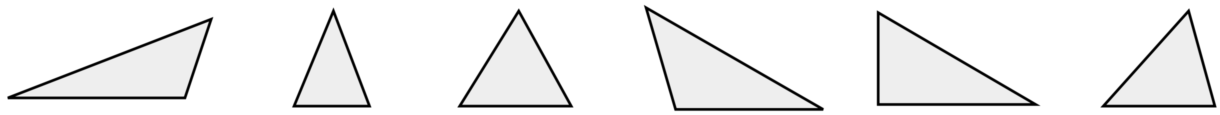
13. Experiment with other lengths. Write a statement about the relationship among the lengths of the three sides of a triangle. ____

____

____

Exploring Types of Triangles

What am I? Scalene? Isosceles? Equilateral? Acute? Right? Or Obtuse?

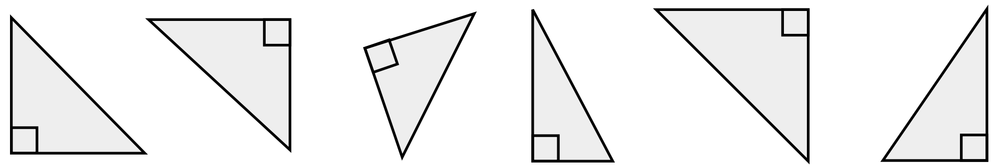


Use Sketchpad to explore more types of triangles.

1. Open the sketch Scalene Triangle (Mac) or Scalene.gsp (Windows).
2. The sketch shows you the angle measures and side lengths. As you drag to answer the questions below, make sure you keep the measures of all three sides different so that the triangle remains scalene.
   a. Can you have a scalene triangle that is also an *acute* triangle? ____________
   b. Can you have a scalene triangle that is also an *obtuse* triangle? ____________
   c. Can you have a scalene triangle that is also a *right* triangle? ____________
3. Open the sketch Isosceles Triangle (Mac) or Iso_tri.gsp (Windows).
4. The sketch shows you the angle measures and side lengths. Drag the figure and answer the following questions.
   a. Can you have an isosceles triangle that is also an *acute* triangle? ____________
   b. Can you have an isosceles triangle that is also an *obtuse* triangle? ____________
   c. Can you have an isosceles triangle that is also a *right* triangle? ____________
5. Open the sketch Equilateral Triangle (Mac) or Equi_tri.gsp (Windows).
6. The sketch shows you the angle measures and side lengths. Drag the figure and answer the following questions.
   a. Can you have an equilateral triangle that is also an *acute* triangle? ____________
   b. Can you have an equilateral triangle that is also an *obtuse* triangle? ____________
   c. Can you have an equilateral triangle that is also a *right* triangle? ____________

7. Use any of the three sketches (if needed) to tell whether each statement below is true *always, sometimes,* or *never.*
   a. An acute triangle is isosceles. ____________________
   b. An obtuse triangle is scalene. ____________________
   c. An obtuse triangle contains a right angle. ____________________
   d. A triangle contains two obtuse angles. ____________________
   e. A right triangle is equilateral. ____________________
   f. An isosceles triangle is equilateral. ____________________
   g. A right triangle has two acute angles. ____________________
   h. An equilateral triangle is isosceles. ____________________

Exploring the Pythagorean Theorem*



1. Open the sketch Pythagoras 1 (Mac) or Pythag1.gsp (Windows).
2. Notice the following features of the figure.
   a. Triangle **TRY** is a right triangle.
   b. The figure constructed on each side of ∆**TRY** is a square.
   c. For each square, the length of each side equals the length of one side of the triangle.
   d. The lengths of the sides of the triangle and squares are shown in the sketch.
   e. The areas of the squares are shown.
3. Using the displayed measures of the areas of the squares, build a table to show the values as you change the triangle.
   a. Hold down the Shift key and click on the displayed measures of $\mathbf{a}^2$, $\mathbf{b}^2$, and $\mathbf{c}^2$.
   b. Go to the **Measure** menu and choose **Tabulate**.
   c. Drag the resulting table of values to a convenient location in your sketch.
   d. Drag **T** or **Y**.
   e. Double click on a value in the table to add a new column.
   f. Repeat steps d and e until you have added several columns.
4. Do you see any relationship among the values in your table? If yes, explain.

   Note: Keep in mind that measurements are rounded to the nearest tenth of a square centimeter.

   ____

   ____

   ____

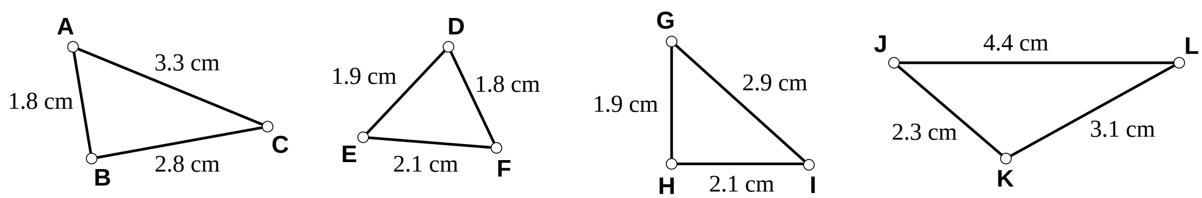
   ____

*There is evidence that special cases of this theorem were known in Babylon (present-day Iran and Iraq) at least 1,000 years before Pythagoras lived. There exists what some consider a proof of this theorem by the Chinese in *Zhoubi suajing,* which may date from before Pythagoras's time. For more details, see Victor J. Katz, *A History of Mathematics,* (HarperCollins ©1995).

If no, do the following: Add **a**^{2} and **b**^{2}. Notice how the sum compares with **c**^{2}. Now state the relationship in the space above.

Tip: To calculate the sum of **a**^{2} and **b**^{2} in your sketch, go to the **Measure** menu and choose **Calculate.** You can use Sketchpad's calculator to find the sum for you.

5. Drag vertices **T** and **Y** to determine whether this relationship always exists.
6. Explain in your own words what this activity tells you about the relationship among the sides of any right triangle.



7. Open the sketch Pythagoras 2 (Mac) or Pythag2.gsp (Windows).

   Notice that the following measures are displayed: the lengths of **a**, **b**, and **c**; the values of **a**^{2}, **b**^{2}, and **c**^{2}; the measure of ∠**NOW**; and the value of **a**^{2} + **b**^{2}.
8. Drag the endpoints of the parallel segments on the right side of your sketch to change the lengths of sides **a**, **b**, and **c** in your sketch. Drag vertices **N** and **W** to change the size of ∠**NOW**.

   Notice that dragging **N** and **W** changes the angle measure of ∠**NOW** and side **c** but not the lengths of **a** and **b**. Also, notice that dragging segments **a** and **b** changes the lengths of **a**, **b**, and **c** but not the measure of ∠**NOW**.
9. Drag **a** and **b** to the designated lengths, and then drag vertex **N** or **W** to change the measure of ∠**NOW** to answer the questions and complete the table.

	Side **a**	Side **b**	Can you drag **N** and/or **W** so that ∠**NOW** is acute and **a**^{2} + **b**^{2} = **c**^{2}?	Can you drag **N** and/or **W** so that ∠**NOW** is obtuse and **a**^{2} + **b**^{2} = **c**^{2}?	Drag **N** and/or **W** until **a**^{2} + **b**^{2} = **c**^{2}. What is the measure of ∠**NOW** when this happens?
a.	3.0 cm	4.0 cm			
b.	1.5 cm	2.0 cm			
c.	0.5 cm	1.2 cm			
d.	6.0 cm	8.0 cm			
e.	3.4 cm	2.4 cm			
f.	3.5 cm	3.5 cm			

10. Use your results from step 9 to answer each question below.
    a. If the sum of the squares of the two short sides ($\mathbf{a}^2 + \mathbf{b}^2$) equals the square of the longest side ($\mathbf{c}^2$) in a triangle, is it *always* a right triangle? ____________
    b. If the sum of the squares of the two short sides ($\mathbf{a}^2 + \mathbf{b}^2$) does not equal the square of the longest side ($\mathbf{c}^2$), can you ever have a right triangle? ____________
    c. Can you have a right triangle with the side lengths shown below? Write yes or no for each.

       20, 21, 29 ____________

       8, 15, 17 ____________

       8, 10, 13 ____________

11. Complete this statement about the relationship of side lengths in right triangles:

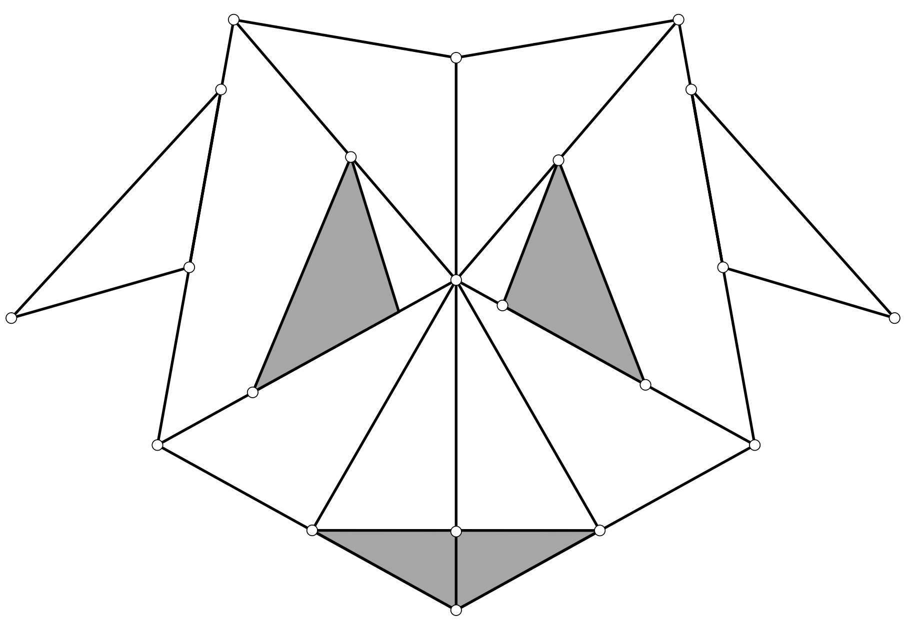
    A triangle will be a right triangle only when ____________________________________

    ____

Triangle Search

1. Open the sketch Triangle Search (Mac) or Tri_srch.gsp (Windows).
2. Being careful not to drag any points or change any shapes in any way, work together to complete the activity: Count the total of each kind of triangle found in the figure below. Be sure to include triangles formed by two or more individual shapes. Use Sketchpad tools and the **Measure** menu to verify lengths of sides and measures of angles when needed.

   Remember: You can measure the length of a side by selecting the side and choosing **Length** from the **Measure** menu. However, when a segment serves as a side to several triangles, you may need to select the two endpoints of the side of your triangle and choose **Distance** from the **Measure** menu. Also, do not add two parts of a side to get its length. Instead, always select the endpoints of the side you wish to measure and choose **Distance** from the **Measure** menu.
3. Put your names on your sketch and print out a copy to hand in.



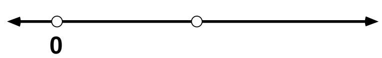
acute triangles ______ scalene triangles ______
obtuse triangles ______ isosceles triangles ______
right triangles ______ equilateral triangles ______
number of different sizes of triangles ______ total number of triangles ______

The Real Real Number Line

In this activity, you will construct a Sketchpad number line that will display some irrational numbers as well as the whole numbers traditionally displayed on a number line.

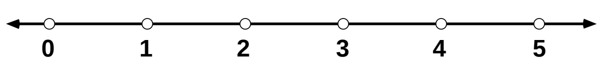
1. Open a new sketch. Go to the **Display** menu and choose **Preferences**. Change the distance unit to Inches if needed.
2. Using the Line tool , draw a horizontal line in your sketch.
3. Hold down the Shift key. Using the Selection Arrow tool , select both points on the line.
4. Go to the **Display** menu and choose **Hide Points**.

5. Select the line. Go to the **Construct** menu and choose **Point On Object**.
6. Using the Text tool , click on the point on your horizontal line to display its label. Double click on the label and change its name to 0 (to mark it as the zero point on your number line).
7. Using the Selection Arrow tool , select the zero point. Go to the **Transform** menu and choose **Translate**. Translate as shown at right.

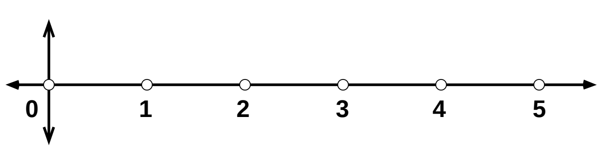


Translate dialog box (Macintosh)

8. Continue selecting a point and translating it until you have a number line at least 5 inches long. Label the points as they appear below.

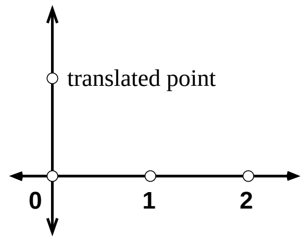


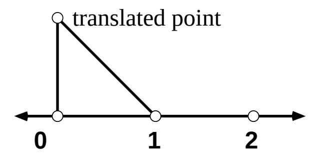
9. Select the horizontal line and the zero point on that line. Go to the **Construct** menu and choose **Perpendicular Line**.



10. Select the zero point and translate it vertically 1 unit.

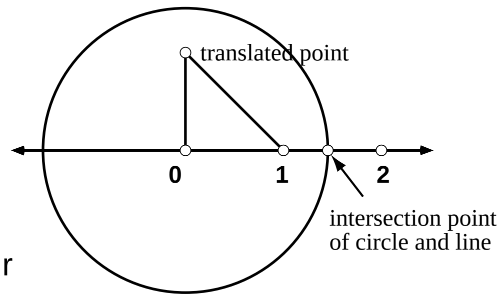
11. Select the point 0, the point 1, and the translated point and construct segments between them to create a right triangle. (You may want to hide the vertical perpendicular line before you do this.)





This is a 1, 1, $\sqrt{2}$ right triangle.

12. Select the hypotenuse and the zero point. Go to the **Construct** menu and choose **Circle By Center+Radius**.



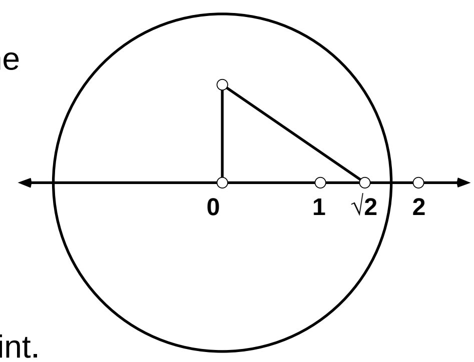
13. Click on the point of intersection of the circle and the number line to construct the point $\sqrt{2}$ on your number line.

    Hide your circle and the hypotenuse. Label this point.

Note: On a Macintosh, type option v to display √.
With Windows, use the Symbol font, then hold down the alt key while you type 0214 on the number keypad.

14. Construct a segment between the translated point and the point $\sqrt{2}$. This construction yields a 1, $\sqrt{2}$, $\sqrt{3}$ right triangle.

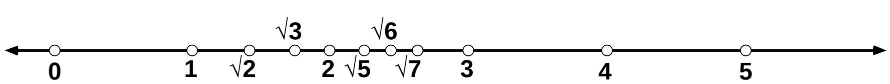
15. Select the hypotenuse and the zero point. Go to the **Construct** menu and chose **Circle By Center+Radius**.



16. Click on the point of intersection of the circle and the number line to construct the point $\sqrt{3}$ on your number line.

17. Hide your circle and the hypotenuse. Label this point.

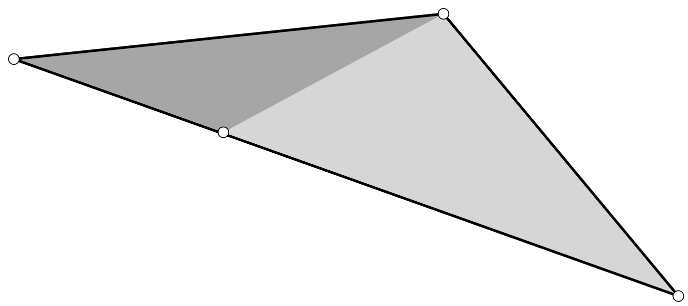
18. Repeat the above process to construct the points $\sqrt{5}$, $\sqrt{6}$, and $\sqrt{7}$ on your number line as shown below. Be sure to hide objects.



19. Print out a copy of your sketch to hand in.

20. Using your pencil, place labeled points on the printed copy of your number line to approximate the numbers $\sqrt{10}$, 2.5, $\sqrt{15}$, 3.75 , $\sqrt{16}$, $\sqrt{18}$, and $\sqrt{24}$.

Slice It, Dice It



1. Use Sketchpad to construct a triangle. Cut the triangle into exactly two triangles.
2. Is it possible for these two smaller triangles to have the same area? Make a conjecture. (It may be helpful to choose **Preferences** from the **Display** menu and set the distance unit to tenths). ______________________________

____

____

____

3. Does your conjecture depend on a particular type of original triangle? Is there just one such pair of triangles? Will the two smaller triangles ever be congruent? Use sketches to help you. Explain your reasoning. ______________________________

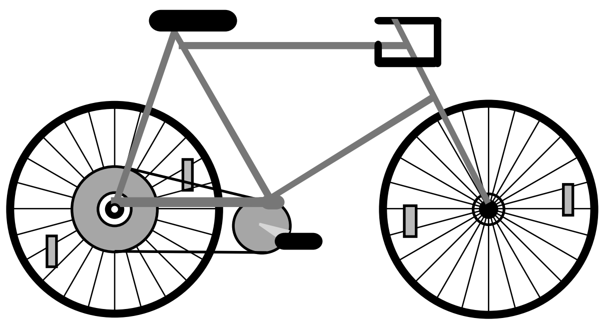
____

____

____

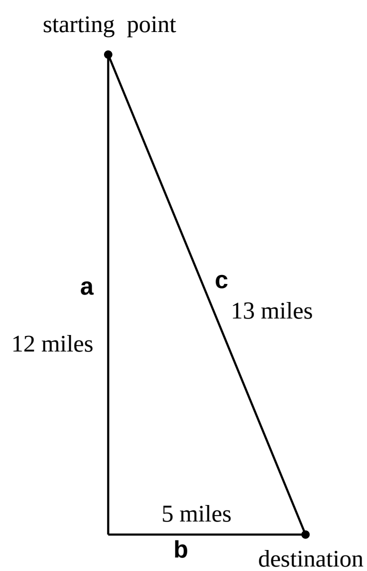
PT Problems

Create a word problem of your own that can be solved by using the Pythagorean Theorem. Put the problem, a drawing, and the solution in a sketch. Put your name on the sketch and print out a copy to hand in.



Student Example:

Emily is riding her bicycle on straight roads 12 miles due south, then 5 miles due east. If Katy starts at the same place and bikes along a straight diagonal road that ends where Emily does, how far must Katy bike?



Example Solution:

In the diagram, **a** = 12 miles, **b** = 5 miles, and the triangle is a right triangle. You must find **c**.

$$\mathbf{a}^2 + \mathbf{b}^2 = \mathbf{c}^2$$
$$5^2 + 12^2 = \mathbf{c}^2$$
$$25 + 144 = \mathbf{c}^2$$
$$169 = \mathbf{c}^2$$
$$13 = \mathbf{c}$$

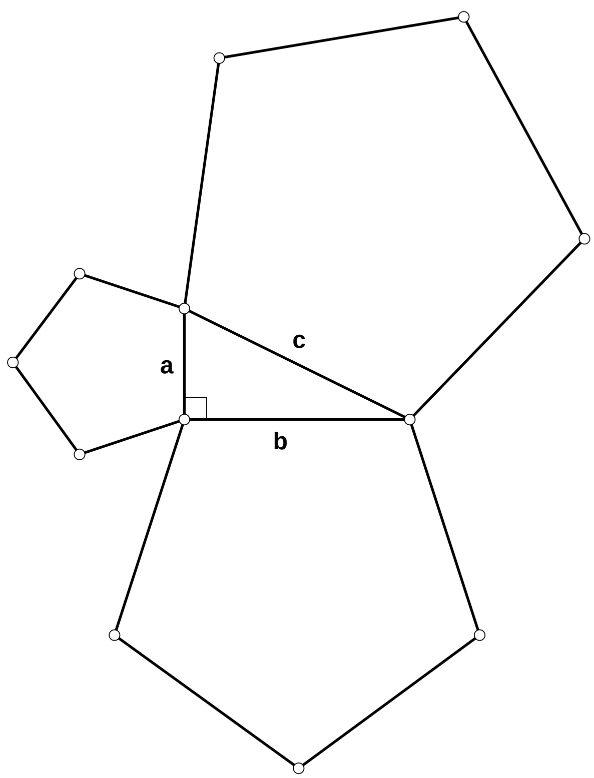
Katy must bike 13 miles.

Pythagoras Plus

Use Sketchpad to explore further with the Pythagorean Theorem. Remember how we used squares on the sides of a right triangle to show the Pythagorean Theorem? Recall that in a right triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides. Do you think this relationship will hold for polygons besides squares? Test the idea.

1. Open a new sketch and construct a right triangle. Next, construct a regular polygon (a polygon with all sides congruent and all angles congruent) of some kind so that the hypotenuse is one side of the polygon. Construct the same kind of regular polygon on each leg of your triangle, using the leg as one side of your polygon. Construct the polygon interiors.

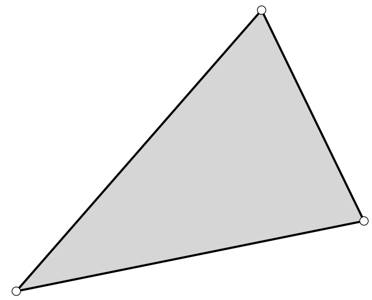
2. Use the **Measure** menu and the Sketchpad calculator to find and display the area of your three polygons. Show whether the area of the figure constructed on side **a** plus the area of the figure constructed on side **b** equals the area of the figure constructed on hypotenuse **c** in your sketch. Print out a copy with all measures and your conclusions displayed. Turn it in to your teacher.



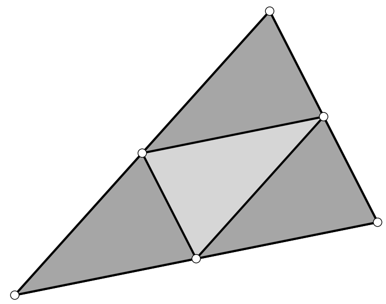
A Look Inside

Use Sketchpad to investigate the effects of subdividing triangles.

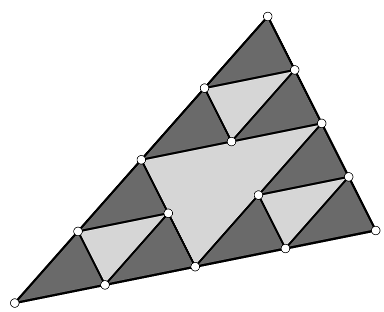
1. Construct a triangle and use **Calculate** in the **Measure** menu to find its perimeter. Call this Stage 0, because this is the figure before any changes are made.



2. For Stage 1, find the midpoint of each side of your triangle and construct four smaller congruent triangles. Find the sum of the perimeters of the three corner triangles.



3. For Stage 2, find the midpoint of each side of your corner triangles and construct four smaller congruent triangles inside each. Find the sum of the perimeters of the nine corner triangles within the original corner triangles.



4. Compare the total perimeters for each stage and look for patterns. Write any conjectures and explain your reasoning. ____

   ____

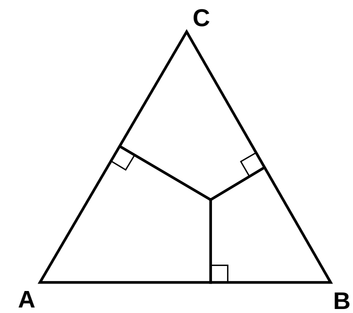
   ____

   ____

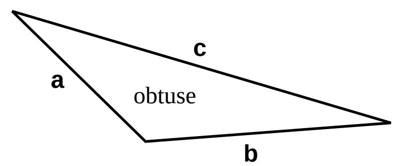
   Test your conjectures by dragging any vertex of the original triangle. You can also create more stages to test your conjectures.

5. Investigate *total area* using the same stages.

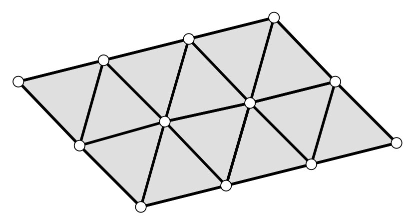
1. Use Sketchpad to construct an equilateral triangle. Place a point anywhere inside the triangle and construct a segment perpendicular to each side of the triangle from that point. Find the sum of the three segments. Drag a vertex of the triangle or your interior point. Make any conjectures you can.



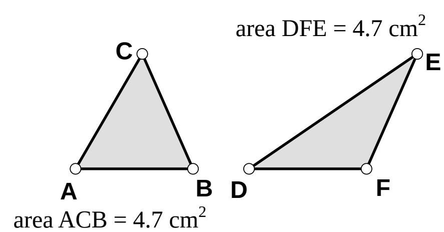
2. The Pythagorean Theorem states that, for every right triangle, the square of the longest side equals the sum of the squares of the other two sides. Use Sketchpad to explore this kind of comparison in acute triangles. How does the square of the longest side compare to the sum of the squares of the other two sides? How do these compare in obtuse triangles?



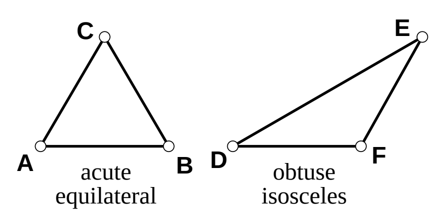
3. Use Sketchpad to explore tessellations with triangles. Try to tessellate the plane with each type of triangle (equilateral, isosceles, scalene, right, acute, and obtuse). What conjectures can you make?



4. Use Sketchpad to construct several large triangles with the same area. Make the shapes as different as you can. (You may want to display altitude, angle measures, side and altitude lengths, and area measure for each triangle.) Print out your sketches, cut out the triangles, and make a mobile.



5. Use Sketchpad to construct several large triangles. Create as many different shapes as you can. Label each triangle according to its sides and according to its angles. Print out your sketches, cut out the triangles, and make a mobile.



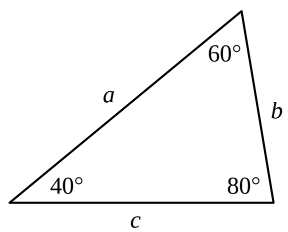
Triangles Wrap-Up

For each set of lengths in questions 1–3, tell whether it is possible to draw a triangle with sides of those measures. Write yes or no.

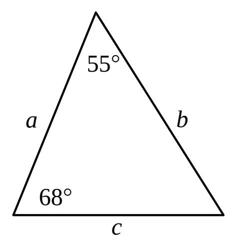
1. 6 cm, 8 cm, 10 cm ______________
2. 3.5 in., 4.5 in., 8 in. ______________
3. $2\frac{3}{4}$ in., $2\frac{1}{4}$ in., 1 in. ______________

For each triangle in questions 4–6, use the labels given to list the sides in order from shortest to longest.

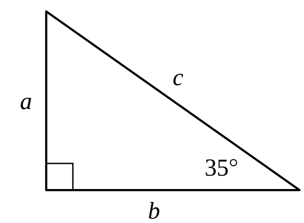
4. ______________



5. ______________

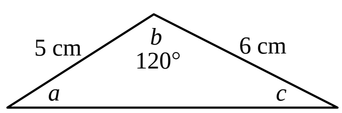


6. ______________

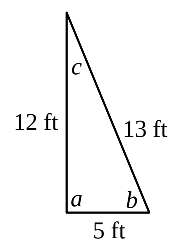


For each triangle in questions 7–9, use the labels given to list the angles in order from smallest to largest.

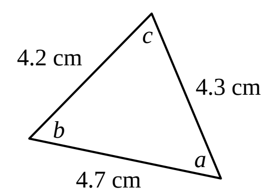
7. ______________



8. ______________



9. ______________



Draw an example of each type of triangle in questions 10–15. If it cannot be done, write *impossible* in the space.

10. acute isosceles triangle

11. obtuse scalene triangle

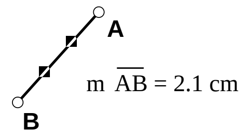
12. right isosceles triangle

13. obtuse equilateral triangle

14. right equilateral triangle

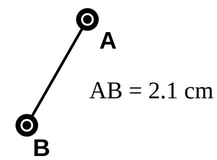
15. acute scalene triangle

To measure the length of a line segment (use the Selection Arrow tool)



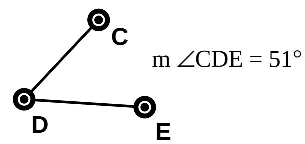
1. Click on the segment.
2. Go to the **Measure** menu and choose **Length**.

To measure the distance between two points (use the Selection Arrow tool)



1. Hold down the Shift key.
2. Click on one of the points and then on the other.
3. Go to the **Measure** menu and choose **Distance**.

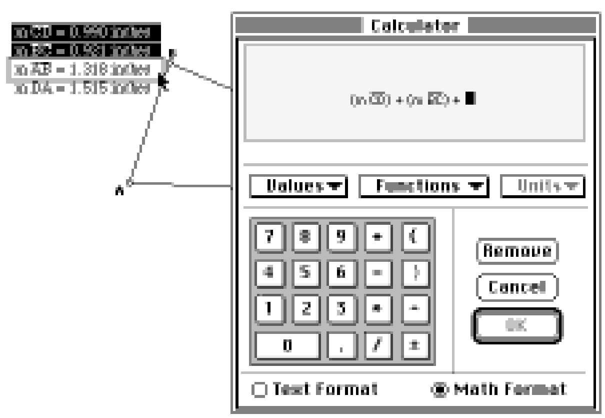
To measure an angle (use the Selection Arrow tool)



1. Hold down the Shift key.
2. Click on three points you could use to name the angle, with the vertex your middle selection.
3. Go to the **Measure** menu and select **Angle**.

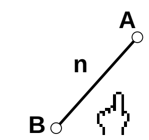
To sum measures (use the Selection Arrow tool)

1. Go to the **Measure** menu and choose **Calculate** or double click on a measure to activate the calculator.
2. Click on one of the measures that is showing in your sketch.
3. Click on the **+** sign and then click on another measure showing in your sketch.
4. Repeat until all the measures you wish to sum appear, then click on **OK.**



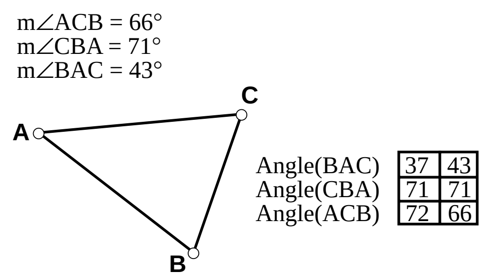
To display a label and move it (use the Text tool)

1. Click on the object. For a segment, click on the segment itself, not on the endpoints.
2. To move the label, click on the label and drag it to any location close to the object.



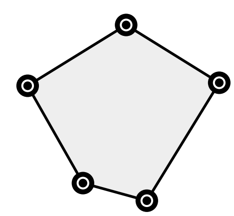
To make a table (use the Selection Arrow tool)

1. Hold down the Shift key and click on the measures you wish to show in your table.
2. Go to the **Measure** menu and choose **Tabulate**.
3. Drag the table of values to a convenient place in your sketch.
4. Drag your object to change its size.
5. Double click on a value in your table to add a new column. Repeat steps 4 and 5 to add new values.



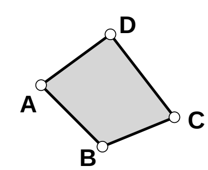
To construct a polygon interior (use the Selection Arrow tool)

1. Hold down the Shift key. Click on the vertices of the polygon in order, either clockwise or counterclockwise.
2. Go to the **Construct** menu and choose **Polygon Interior**.



To measure perimeter or area of a polygon (use the Selection Arrow tool)

1. Construct the polygon interior, if you haven't done so already.
2. Click on the polygon interior.
3. Go to the **Measure** menu and choose **Perimeter** or **Area**.



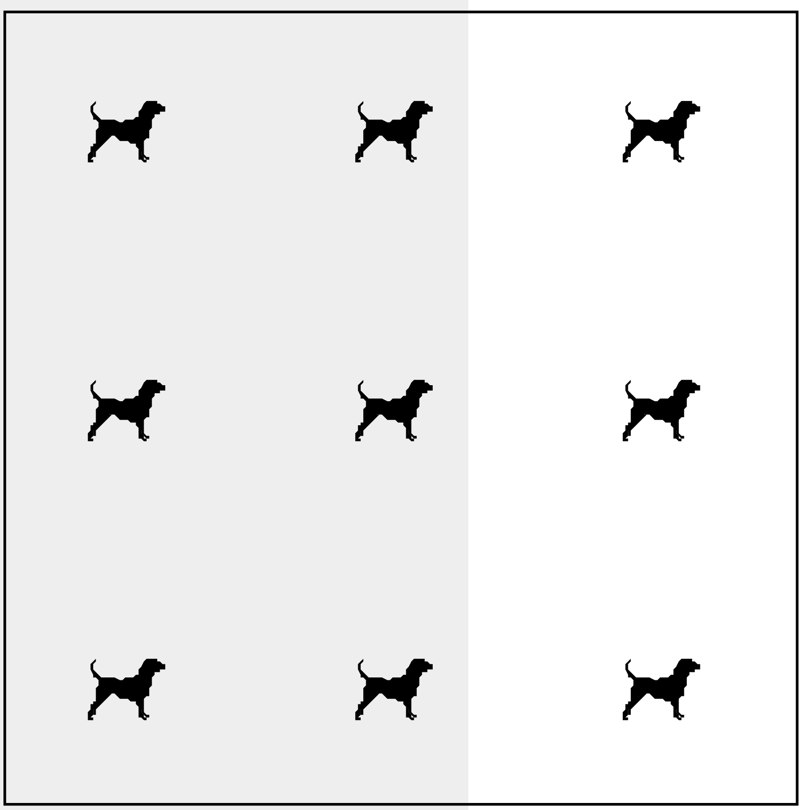
area ABCD = 2.8 cm^2

To write your name(s) on a sketch and then print (use the Text tool)

1. Drag in a blank area in your sketch to create a text box.
2. Type your name(s) in this box.
3. Go to the **File** menu and choose **Print**.

Unit 4

QUADRILATERALS



Commentary

Teaching quadrilaterals had always been difficult for us. Year after year, students took notes faithfully, did well on a quiz when asked to list the characteristics of each type of quadrilateral, “played” diligently with shapers (cardboard strips used to help clarify characteristics), and then (ugh!) did poorly on the unit test.

With The Geometer’s Sketchpad, the experience was quite different. After a day of introductory work in the classroom demonstrating each type of quadrilateral with Sketchpad and an overhead projector, we went to the computer lab, where students completed the activity **Exploring Quadrilaterals,** working in pairs. There we discovered the importance of demonstrating how to test a pair of sides or angles for congruence—because it had been a while since we had worked in the lab, most of the kids had forgotten how to do that with Sketchpad.

If your students are at a level where they would benefit from exploring the concept of simple closed figures, the activity **Simple Closed Figures** is an appropriate way to introduce them to the dynamic figures that are created using Sketchpad.

In **Exploring Quadrilaterals,** students should discover the properties of various types of quadrilaterals. They make these discoveries by playing with sketches that are constrained to remain a certain shape. For example, the parallelogram will remain a parallelogram no matter how the students drag its vertices. When a figure such as the parallelogram can be made into a square by dragging, it is easy for most students to conclude that a parallelogram is sometimes a square. By measuring the sides and angles, some kids discover that this happens whenever a pair of adjacent sides is congruent and forms a right angle. While the kids worked, we overheard conversations between and among pairs: “Do we need to measure all the sides of the rhombus? I can tell they’re all the same” “Hey, did you guys get the trapezoid to turn into anything?” and “If we got the rectangle to be a square, does that mean a rectangle is always a square or that a square is always a rectangle?” It seemed as if most of the kids were really grasping the properties of each type of quadrilateral and the relationships between the types.

As we went over the activity in class the next day, we were excited to see how clearly the kids seemed to understand the material. A few were shaky on the *sometimes/always/never* questions, but the problem seemed to lie with the format of the questions rather than with their understanding of the properties. We still believe it is important to use this format because of the extra thinking it requires. Students get better at it with practice.

Properties of Quadrilaterals is included as an open-ended alternative to **Exploring Quadrilaterals.** It can be used as a follow-up or an assessment instrument as well.

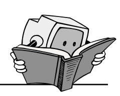
Diagonals in Quadrilaterals is appropriate only for students who can clearly differentiate among the types of quadrilaterals. Before starting it, we found it very important to go over Sketchpad skills such as constructing the point of intersection for the diagonals and then a new segment for each part. You may want to point out to students that the measure of the length of a segment is the same as the distance between its two endpoints. Since measuring the distance between two points is, in this situation, a simpler maneuver with Sketchpad, you may want to encourage your students to use this method. During this activity, students test the diagonals of several kinds of quadrilaterals to determine whether the diagonals are always congruent, perpendicular, and/or bisectors of each other. Some students need to be prodded to have Sketchpad actually measure angles and segments; they often want to “eyeball” the sketch and guess at the answers. Student comments during the activity made it clear that manipulating the figures in the sketches enabled many of them to see some of the properties clearly for the first time. Back in the classroom, our impressions of their overall understanding were verified when their performance on the unit test was the best ever.

We found **Diagonals Inside and Out** to be especially effective when used in response to questions raised by students about convex versus concave polygons. (Of course, as all middle-school teachers know, cleverly guided dialogue by you can often provoke such questions!) Similarly, **Midpoint Quadrilaterals** is most meaningful as a search for answers to "what if" questions: "What happens if you connect the midpoints of the sides of a quadrilateral to construct another quadrilateral inside the original figure?"

Dog Pens lends itself to a cooperative-learning format. Middle-school kids enjoy creating a Sketchpad illustration to show their solution.

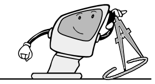
Quadrilateral Puzzle is harder than it first appears. Students should have done some constructions before being assigned this activity. Doing **Constructing a Rhombus** and/or **Constructing a Parallelogram** (see the Constructions unit) is desirable. If students have made scripts for constructing quadrilaterals, these would also be appropriate here. Because it is easy to overlook some quadrilaterals when making a key, we found that requiring checks by other students *before* the puzzle is handed in helped reduce student errors.

Prerequisite Mathematical Terms and Concepts



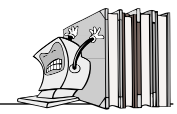
- *polygon, quadrilateral, opposite sides, opposite angles, consecutive sides, consecutive angles, parallel, perpendicular*
- *congruent sides, congruent angles*

Recommended Sketchpad Proficiency



- Basic knowledge of the freehand tools.
- Use of the Polygons Quick Reference Guide (in the Triangles unit) is suggested.

Essential Vocabulary



Bisect—to divide into two congruent parts (e.g., if a point divides a segment into two congruent parts, it bisects the segment; if a ray divides an angle into two congruent angles, it bisects the angle)

Concave polygon—a polygon containing at least one interior angle with measure greater than 180°

Convex polygon—a polygon in which the measure of each interior angle is less than 180°

Diagonal—a segment connecting any two nonconsecutive vertices of a polygon

Intersection point—any point that two geometric shapes have in common

Parallelogram—a quadrilateral with two pairs of parallel sides

Polygon—a simple closed figure in a plane, with segments as sides

Quadrilateral—a polygon with four sides

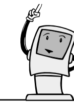
Rectangle—an equiangular parallelogram

Rhombus—an equilateral parallelogram

Square—an equilateral rectangle

Trapezoid—a quadrilateral with exactly one pair of parallel sides

Instructions for Teacher Demonstrations



In each unit, we have included sketches and guidelines for teacher demonstrations designed as introductions by the teacher in whole-class presentations before students attempt the various activities independently. They include the introduction of relevant mathematical vocabulary and concepts as well as appropriate Sketchpad skills. Often there are one or several activities that you will feel comfortable assigning to your students with very little guidance beforehand. In this unit, for example, **Simple Closed Figures** works fine as an exploratory introduction to quadrilaterals and requires very little technical expertise. On the other hand, one or more activities in each unit are designed to maximize guided discovery by the students. For these activities, a careful teacher demonstration will focus the students' attention on targeted mathematical concepts and prevent distractions due to lack of Sketchpad proficiency. In this unit, **Exploring Quadrilat-erals** is designed to follow such a demonstration; the instructions are included later in these notes. Of course, you may want to use portions or adaptations of the demonstration before other activities; certainly, you will want to preview activities to be sure they are appropriate for the mathematical and Sketchpad skill levels of your students.

You may find the Polygons Quick Reference Guide (at the end of the Triangles unit) helpful in preparing for the quadrilaterals demonstrations.

Exploring Quadrilaterals

1. Discuss quadrilaterals. Open a new sketch, draw a general quadrilateral, and demonstrate the following Sketchpad skills:
   - Set preferences.
     a. Choose **Preferences** from the **Display** menu.
     b. Select the following settings:
        Distance Unit—cm
        Angle Unit—degrees
        Precision—hundredths
        Precision—units
   - Use the Text tool to change the name for a measure (a name often needs shortening).
2. Explain that the figure in each student sketch will remain that kind of quadrilateral when it is dragged. [We suggest opening the sketch Rhombus (Mac) or Rhombus.gsp (Windows) and dragging to show that it can become a square. Point out that the figure is still a rhombus.]
3. Using a general quadrilateral sketch, demonstrate and discuss the following Sketchpad skills:
   - Test sides and angles for congruence.
   - Find the sum of the angle measures in a quadrilateral using **Calculate** in the **Measure** menu.
   - Drag a figure to determine relationships ranging from "no relationship" to "all *x* are always congruent."
   - Drag a figure to test whether it can become another kind of quadrilateral (e.g., drag a rectangle to a square).
   - Drag a figure to determine *sometimes/always/never* for statements in the form "A type *c* quadrilateral is *sometimes/always/never* a type *d* quadrilateral." Solicit student suggestions for possible statements.
4. Discuss *diagonals, intersection point, bisect.* Demonstrate the following Sketchpad skills using a general quadrilateral sketch:
   - Construct the diagonals of a quadrilateral.
   - Construct the intersection point of diagonals.
   - Measure the length of part or all of a diagonal.
   - Measure angles formed at the intersection point of diagonals.
   - Determine whether two diagonals are perpendicular.

5. Using the various types of quadrilateral sketches in the Quadrilateral folder (Mac) or Quads directory (Windows), discuss dragging.
   - Test diagonals for a particular kind of quadrilateral from the sketch.
   - Determine whether a statement is *always* true for a particular kind of quadrilateral. Solicit student suggestions for possible statements.

Note to Teacher: **Constructing a Rhombus** and **Constructing a Parallelogram** (in the Constructions unit) fit nicely with this unit.

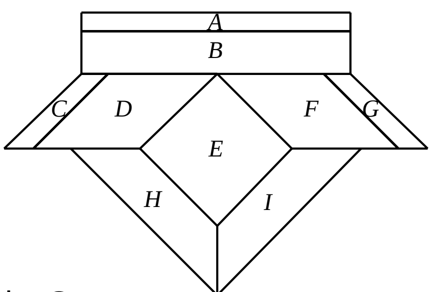
Writing Prompts

Ask students to choose a topic below and write a short paper, including sketches created with Sketchpad.

- Introducing the Quadrilateral Family
- A Day in the Life of Quadra Lateral
- Diagonals in Quadrilaterals
- Questions I Have About Quadrilaterals

Example of Student Work

Quadrilateral Puzzle



by Scott

Key

Trapezoids (2)	*H, I*	2 pts.
Squares (1)	*E*	4 pts.
Rectangles (3)	*A, B, A+B*	9 pts.
Rhombuses (2)	*D, F*	6 pts.
Parallelograms (4)	*C, G, C+D, F+G*	8 pts.
	Total points:	29

Answers for Exploring Quadrilaterals Activity

2. a. all equal b. all equal c. always
   d. always e. always
4. a. all equal b. Two pairs of opposite angles have equal measures.
   c. always d. always e. always
   f. always g. sometimes h. always
6. a. Two pairs of opposite sides have equal lengths. b. all equal
   c. always d. always e. always
   f. sometimes g. always h. sometimes
8. a. Two pairs of opposite sides have equal lengths.
   b. Two pairs of opposite angles have equal measures.
   c. always d. always e. always
   f. sometimes g. always h. always

10. a. no relationship b. no relationship c. yes
d. yes e. no f. always
g. never

12. a. no b. no c. four-sided polygon

Solution for Dog Pens Activity

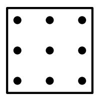
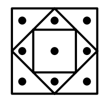


Diagram in sketch Dog Pens



Solution

Answers for Quadrilaterals Wrap-Up

1. a, c, d, e, g, h
parallelogram
2. a, c, e, h
rectangle
3. b, f
trapezoid
4. a, h
square
5. a, g, h
rhombus
6. a, h
square

For questions 7 and 8, answers will vary.

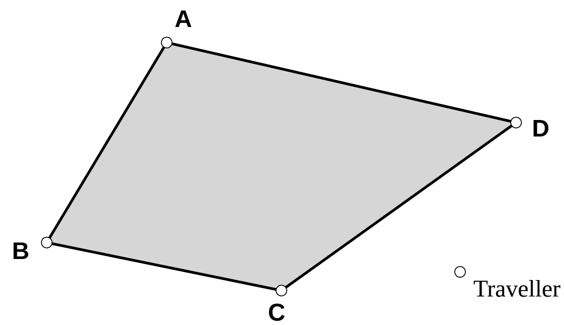
Simple Closed Figures

A *polygon* is a simple closed figure in a plane with segments as sides.

A *quadrilateral* is a polygon with exactly four sides.

Use Sketchpad to explore quadrilaterals as simple closed figures.

1. Open the sketch Quads Intro (Mac) or Quadintr.gsp (Windows).
2. A point can move around a *simple closed figure* in a plane tracing a complete path that doesn't cross itself. Animate Traveller and observe the path traced on quadrilateral **ABCD**. Is the quadrilateral a simple closed figure? Explain.

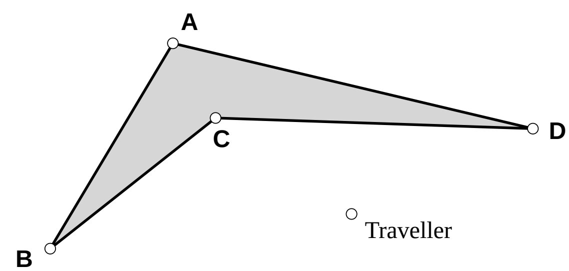


____

____

____

3. Drag any vertex of quadrilateral **ABCD** until it is concave (i.e., until one of the interior angles is greater than 180°). Animate Traveller and observe the path traced. Is the figure a simple closed figure? Explain. Is the figure still a quadrilateral? Explain.

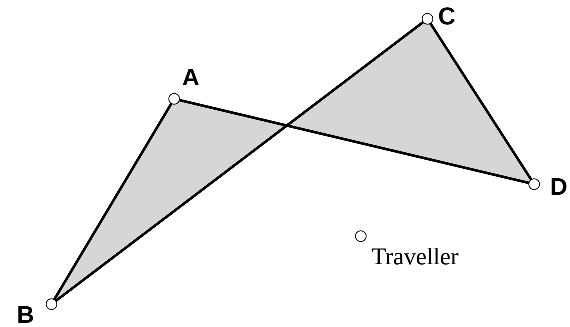


____

____

____

4. Drag any vertex of quadrilateral **ABCD** until two sides cross. Animate Traveller and observe the path traced. Is the figure a simple closed figure? Explain. Is the figure still a quadrilateral? Explain.

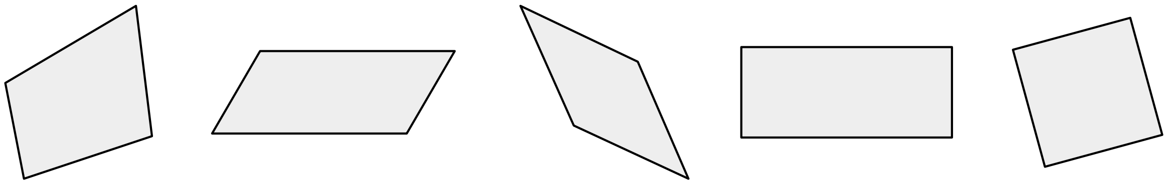


____

____

____

Exploring Quadrilaterals



Use Sketchpad to explore some properties of quadrilaterals.

1. Open the sketch Square (Mac) or Square.gsp (Windows).
2. Follow the instructions in the sketch. As you drag the figure, answer the following questions.
   a. How are the lengths of the sides related? ______
   b. How are the measures of the angles related? ______

   Answer *sometimes, always,* or *never* for each statement about any square.

   c. A pair of opposite sides are parallel. ______
   d. Both pairs of opposite sides are parallel. ______
   e. All angles are right angles. ______
3. Open the sketch Rhombus (Mac) or Rhombus.gsp (Windows).
4. Follow the instructions in the sketch. As you drag the figure, answer the following questions.
   a. How are the lengths of the sides related? ______
   b. How are the measures of the angles related? ______

   Answer *sometimes, always,* or *never* for each statement about any rhombus.

   c. A pair of opposite sides are parallel. ______
   d. Both pairs of opposite sides are parallel. ______
   e. A pair of opposite angles are congruent. ______
   f. Both pairs of opposite angles are congruent. ______
   g. A rhombus is a square. ______
   h. A square is a rhombus. ______
5. Open the sketch Rectangle (Mac) or Rectangl.gsp (Windows).

6. Follow the instructions in the sketch. As you drag the figure, answer the following questions.
   a. How are the lengths of the sides related? ____________________
   b. How are the measures of the angles related? ____________________

   Answer *sometimes, always,* or *never* for each statement about any rectangle.
   c. A pair of opposite sides are parallel. ____________________
   d. Both pairs of opposite sides are parallel. ____________________
   e. All angles are right angles. ____________________
   f. A rectangle is a square. ____________________
   g. A square is a rectangle. ____________________
   h. A rhombus is a rectangle. ____________________
7. Open the sketch Parallelogram (Mac) or Para_grm.gsp (Windows).
8. Follow the instructions in the sketch. As you drag the figure, answer the following questions.
   a. How are the lengths of the sides related? ____________________
   b. How are the measures of the angles related? ____________________

   Answer *sometimes, always,* or *never* for each statement about any parallelogram.
   c. A pair of opposite sides are parallel. ____________________
   d. Both pairs of opposite sides are parallel. ____________________
   e. A pair of opposite angles are congruent. ____________________
   f. A parallelogram is a rectangle. ____________________
   g. A rhombus is a parallelogram. ____________________
   h. A square is a parallelogram. ____________________
9. Open the sketch Trapezoid (Mac) or Trapezd.gsp (Windows).
10. Follow the instructions in the sketch. As you drag the figure, answer the following questions.
    a. How are the lengths of the sides related? ____________________
    b. How are the measures of the angles related? ____________________
    c. Can you drag the trapezoid so that one pair of opposite sides is congruent?

    ____
    d. If the answer to question 10c is yes, are two angles formed in the trapezoid congruent? ____________________

e. Can you drag the figure so that the trapezoid becomes a parallelogram?

____

Answer *sometimes, always,* or *never* for each statement about any trapezoid.

f. One pair of opposite sides is parallel. ____________________

g. Both pairs of opposite sides are parallel. ____________________

11. Open the sketch Quadrilateral (Mac) or Quad.gsp (Windows).

12. Follow the instructions in the sketch. As you drag the figure, answer the following questions.

a. Are any side lengths always equal? ____________________

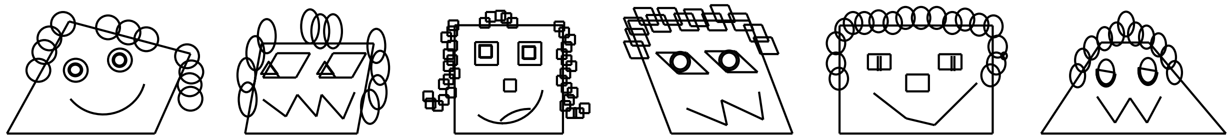
b. Are any angle measures always equal? ____________________

c. What characteristic(s) apply to *all* quadrilaterals? ____________________

____

____

Properties of Quadrilaterals



Queenie Quadrilateral | Penelope Parallelogram | Sarah Square | Raphael Rhombus | Ronnie Rectangle | Timmy Trapezoid

Open all of the sketches from the appropriate list below.

Macintosh:	Windows:
Quadrilateral	Quad.gsp
Parallelogram	Para_grm.gsp
Rectangle	Rectangl.gsp
Rhombus	Rhombus.gsp
Square	Square.gsp
Trapezoid	Trapezd.gsp

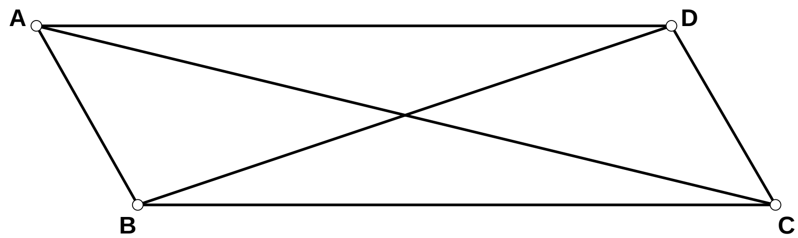
1. List all the characteristics you can for each type of quadrilateral. Organize your information into a chart or table. Your table might be similar to the one shown here.

Type of quadrilateral	Characteristics
Quadrilateral	
Parallelogram	
Rectangle	
Rhombus	
Square	
Trapezoid	

2. Write the simplest *complete* definition you can for each type of quadrilateral. You may find more than one definition for some types.

Type of quadrilateral	Definition
Quadrilateral	
Parallelogram	
Rectangle	
Rhombus	
Square	
Trapezoid	

Diagonals in Quadrilaterals



Use Sketchpad to explore the diagonals in several kinds of parallelograms.

1. Open the sketch Parallelogram (Mac) or Para_grm.gsp (Windows).
2. Construct the diagonals of the parallelogram.
3. Construct the intersection point of the two diagonals.
4. Measure the length of each diagonal.
5. Drag the parallelogram and then answer the following questions.
   a. Are the lengths of the diagonals in a parallelogram always related in the same way? ______________________________
   b. How are the angles formed by the intersection of the diagonals in a parallelogram related? ______________________________

      ____
   c. List any types of parallelograms in which the diagonals are always congruent.

      ____
   d. List any parallelograms in which all four angles formed by the intersection of the diagonals are congruent.

      ____
   e. List any parallelograms in which the diagonals are always perpendicular.

      ____
   f. List any parallelograms in which the diagonals are always congruent and perpendicular.

      ____
   g. Measure the two parts of each diagonal (remember that the measure of the length of a segment is the same as the distance between its two endpoints). List any parallelograms in which the diagonals always bisect each other.

      ____

6. Drag the parallelogram (if needed) to complete the chart below. Write yes or no in each box.

	Diagonals always congruent?	Diagonals always bisect each other?	Diagonals always perpendicular?
Parallelogram			
Rectangle			
Rhombus			
Square			

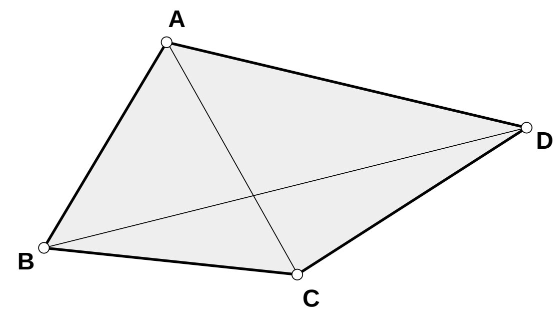
Extension: Open the sketch Trapezoid (Mac) or Trapezd.gsp (Windows). Construct the diagonals. Drag to explore. Write *sometimes, always,* or *never* for each statement below.

1. The diagonals of a trapezoid are congruent. ______________________________
2. The diagonals of a trapezoid bisect each other. ___________________________
3. The diagonals of a trapezoid are perpendicular. ___________________________

Diagonals Inside and Out

1. Open the sketch **Quads & Diagonals** (Mac) or **QuadDiag.gsp** (Windows).

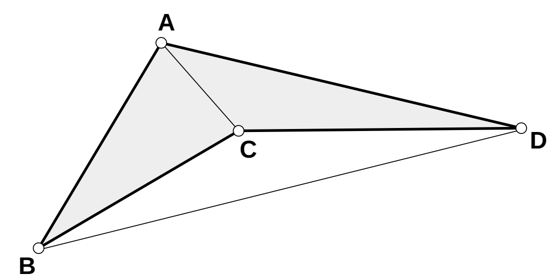
2. Drag any vertex of quadrilateral **ABCD**, being careful to keep the figure convex (i.e., all interior angles are less than 180°). Watch the diagonals as you drag the vertex. Describe the locations of the diagonals with respect to the interior and exterior of the quadrilateral.



____

____

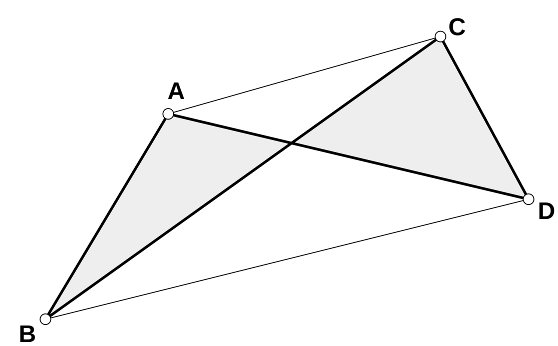
3. Drag any vertex of quadrilateral **ABCD** until it is concave (i.e., one of the interior angles is greater than 180°). Describe the locations of the diagonals with respect to the interior and exterior of the quadrilateral.



____

____

4. Drag any vertex of quadrilateral **ABCD** until two sides cross. Is the figure still a quadrilateral? Explain. Describe the locations of the diagonals with respect to the interior and exterior of the quadrilateral.



____

____

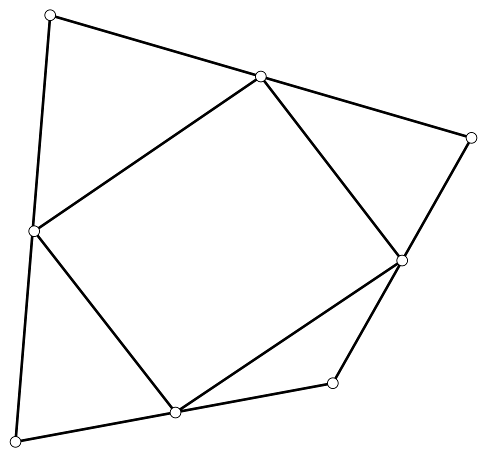
5. Summarize the locations of the two diagonals of figure **ABCD** when it is convex, concave, and not a simple closed figure.

____

____

Midpoint Quadrilaterals

1. Construct a quadrilateral. Construct the midpoints of the sides and connect them to construct another quadrilateral inside your original one. We will call the inside figure the *midpoint quadrilateral.*
2. Drag any vertex of the original quadrilateral and compare the area of your midpoint quadrilateral to the area of your original quadrilateral.
3. Make conjectures based on your findings.



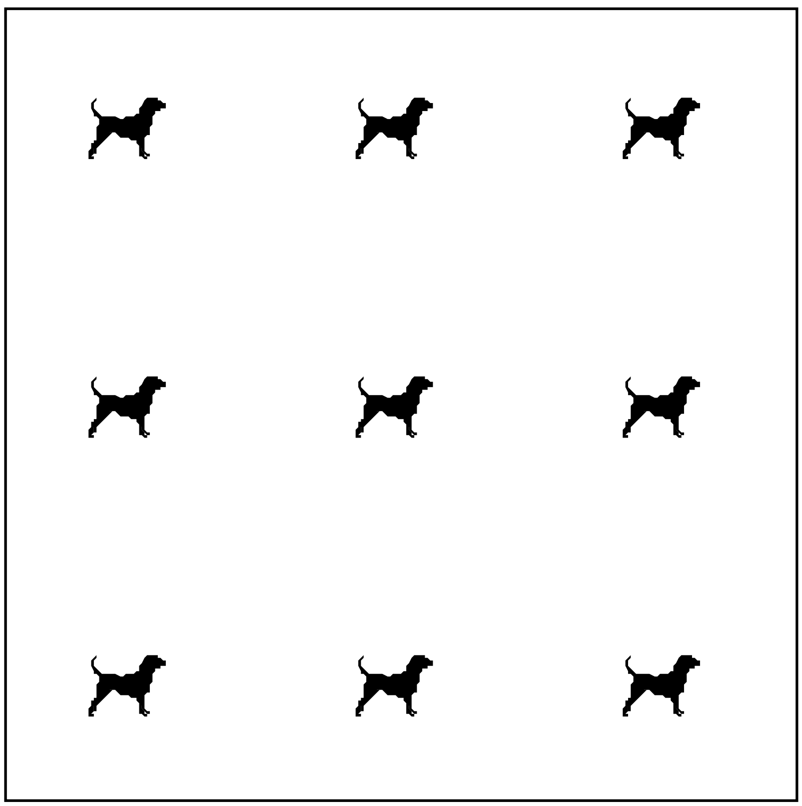
4. What is the shape of the midpoint quadrilateral? Write any conjectures and explain your reasoning.

5. Repeat steps 1–4 for triangles. Write your conjectures about midpoint triangles.

Dog Pens

1. Open the sketch Dog Pens (Mac) or Dog_Pens.gsp (Windows).
2. Working in a group, create a sketch in Sketchpad that solves the problem below.
3. Put your names on your sketch and print out a copy to hand in.

Mr. K. Nine has a square lot for his dogs. He wants to place the dogs as shown below, and he wants a separate pen for each animal. He calls a fencing company and asks them to build two new *square* fences on the lot so that no two dogs share the same pen. Create a sketch for the fencing company to use as a plan.



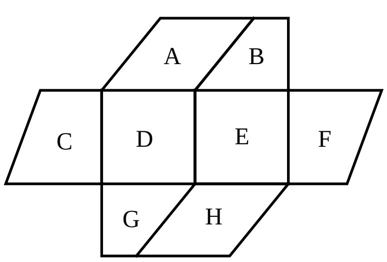
Quadrilateral Puzzle

1. Working with a partner, create an attractive design in which each individual polygon in your design is a quadrilateral. Check your angle measures and side lengths to be sure you include at least one of each kind of quadrilateral: parallelogram, rhombus, square, rectangle, and trapezoid. If a figure needs parallel or perpendicular sides, be sure to use the **Construct** menu. Remember: Simple is elegant.
2. Now for the hard part: Your design must have a value of less than 50 points, with the value of your design determined as shown below. Give only one value to each shape (the highest number of points possible). For example, don't count a square as both a square and a rectangle. Count it only as a square, since a square is worth more points than a rectangle. *Be sure to include shapes formed by two or more of your individual quadrilaterals when you compute the value!*

   Use the following point values:
   each trapezoid = 1 point
   each parallelogram = 2 points
   each rhombus or rectangle = 3 points
   each square = 4 points
3. Include a key for the value of your design. The key should include the total number and value of *each kind* of quadrilateral as well as the overall value. Check your key by letting other students try your puzzle. (See the example below.)

Bonus: You may earn bonus points if you can create a design worth between 25 and 30 points.

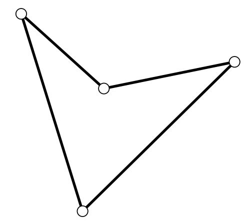
Example:



Key

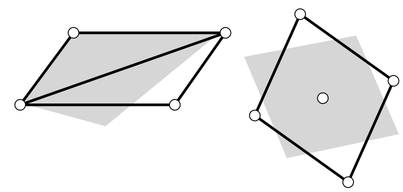
Trapezoids (10)	B, C, F, G, A+B, G+H, C+D, C+D+E, E+F, D+E+F	10 pts.
Squares (2)	D, E	8 pts.
Rectangles (1)	D+E	3 pts.
Rhombuses (2)	A, H	6 pts.
Parallelograms (1)	C+D+E+F	2 pts.
	Total points:	29

1. Use Sketchpad to explore concave quadrilaterals. For example, create sketches to help you explore the sum of the interior or exterior angles in concave quadrilaterals.

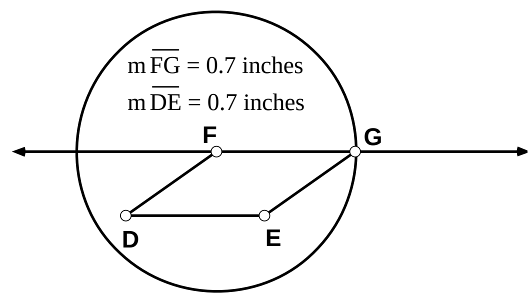


   Note: Sketchpad only displays angle measures less than 180°. When an interior in a concave quadrilateral measures greater than 180°, you will have to use the measure Sketchpad displays to calculate the true angle measure. For angles greater than 180°, the true angle measure is 360° minus the displayed angle measure.

2. Use Sketchpad to explore symmetry in quadrilaterals. Create sketches to demonstrate whether each type of quadrilateral *sometimes, always,* or *never* has reflection or rotation symmetry.

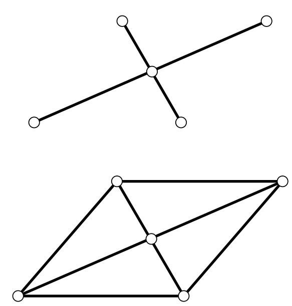


3. Construct a particular type of quadrilateral by focusing on its properties. First, choose one or more properties you feel might be sufficient to define the figure. Then construct the figure using Sketchpad, making sure the figure has the property or properties. Finally, test your figure by dragging to see whether it will always remain that type of quadrilateral. If it does appear to remain that type of quadrilateral, then list the property or properties used to construct it. Repeat for other types of quadrilaterals.

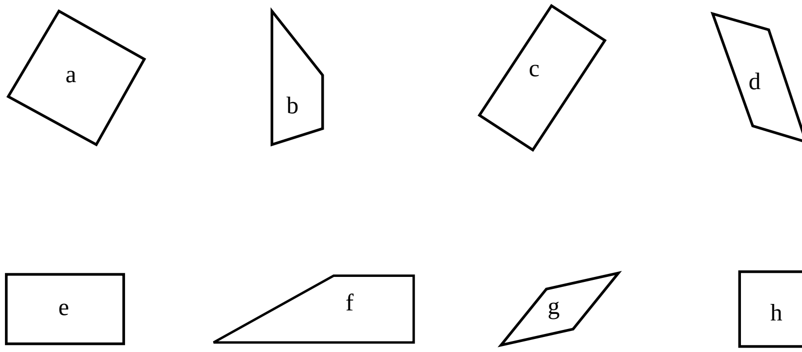


Properties: A pair of opposite sides are congruent and parallel.

4. Construct a quadrilateral starting with a pair of diagonals that are related in some way. For example, construct a pair of congruent and perpendicular segments. Then construct the quadrilateral that has these segments as diagonals. Or start with diagonals that are perpendicular but not congruent. Use Sketchpad to create sketches to help you determine how to construct each type of quadrilateral from its diagonals.



Quadrilaterals Wrap-Up



For questions 1–6, read each set of properties and list the letter for each quadrilateral above that has all those properties. Then give the name of that kind of quadrilateral.

1. Two pairs of parallel sides
   Opposite angles congruent
   Opposite sides congruent
   Figures: ________________________
   Name of quadrilateral: ____________

2. Adjacent sides perpendicular
   Opposite sides parallel
   Figures: ________________________
   Name of quadrilateral: ____________

3. Exactly one pair of parallel sides
   Figures: ________________________
   Name of quadrilateral: ____________

4. All sides congruent
   All angles congruent
   Figures: ________________________
   Name of quadrilateral: ____________

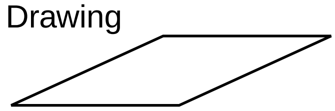
5. Opposite angles congruent
   All sides congruent
   Figures: ________________________
   Name of quadrilateral: ____________

6. Diagonals perpendicular
   Diagonals congruent
   Figures: ________________________
   Name of quadrilateral: ____________

For questions 7 and 8, choose any property a quadrilateral may have. List the letters of the quadrilaterals above that have that property, and draw another quadrilateral with that same property.

Example: Property: All sides congruent
Figures: a, g, h

Drawing



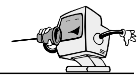
7. Property: ______________________
   Figures: ________________________
   Drawing

8. Property: ______________________
   Figures: ________________________
   Drawing

Unit 5

SYMMETRY

Commentary



One of the rewards of including a symmetry unit in the middle-school mathematics curriculum is that the students are usually unfamiliar with the concepts, so they are fresh and eager to learn. Of course, this unfamiliarity also means we have to proceed slowly as we introduce vocabulary and mathematical ideas to our students for the first time.

Mirrors and Reflection is a simple exploratory introduction to reflections. Our students enjoy dragging vertices of the preimage or image or the mirror to observe the effects.

Our kids truly seem to enjoy **Reflection and Rotation Symmetry,** and most are surprised at the results for at least one of their figures. Many otherwise savvy kids will declare confidently that a regular pentagon has one line of symmetry. Their mouths literally drop open when they find they were mistaken. Lots of furrowed brows turn into “ahas” and smiles by the end of class.

The activities **Diagonals in Parallelograms, Midsegments in Parallelograms,** and **Rotation Symmetry in Parallelograms** are great examples of the adage “Seeing is believing.” Kids (and sometimes teachers!) are amazed when the diagonals of a parallelogram or rectangle are not lines of symmetry. Similarly, many expect midsegments of all parallelograms to serve as lines of symmetry and are surprised when an angle of 90° does not work as an angle of rotation for all parallelograms. Using Sketchpad to discover what works for which parallelograms imprints the relationships in our students’ memories far better than any list, table, or static drawing ever has.

The S Files helps students notice details about figures that alter their symmetries. While this activity may not be appropriate for beginners, we feel it has helped improve the performance of our students on standardized tests because these tests often contain questions involving such figures.

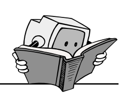
Kids find **Alphabet Symmetry** lots of fun. The extension promotes creativity, and students love to share their designs.

Symmetric Patterns in a Hexagon is also popular. Some kids really get carried away with it, while others struggle a bit at first. Being shown a sample solution helps some students get started. We have seen much improvement in kids’ visual skills simply as a result of doing this project themselves and testing the sketches of other students.

The highlight of this unit for our kids each year is the project **Make Me Symmetric.** This project has been successful at our school with all grade and skill levels. We have found that working in pairs works very well for this project. Every year, both the students and teachers are astounded at its overwhelming success. Students make terrific gains in both their mathematical skills and their self-confidence. A phenomenon that shouldn’t surprise us is that some of the most creative projects are done by students who don’t usually lead the math class. In fact, the poorest calculator or logician often has the best project! It is also gratifying that someone like John G. seems to emerge in every class—a kid who gets so enthralled that he becomes the “Animation Expert” and thrives on helping classmates achieve the effects they desire in particular sketches. The students are always very proud of the products they are able to create, and rightly so. They often come back to visit our classrooms, even a couple of years later, to remind us of the working windshield wipers on the animated car they made or the reaction of their classmates when bees danced across the screen in their sketch. “I had no idea I could draw like that!” they often exclaim when looking at their finished product. The mathematics they learn while doing this project amazes us as well.

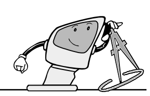
Be sure to cover the **Animation Tours,** found in the Tours unit, before the students start their projects. A little time spent by students in learning the basics of animation will go a long way toward producing confidence and daring. It also prevents their asking you the same questions a multitude of times. A teacher demonstration that includes examples of student symmetry projects is a good motivator to get things off to a solid start. You will need to emphasize the fact that you cannot animate a point that defines an object on that object. For example, you cannot animate the endpoint of a segment along the segment; instead, you have to animate some other point along the segment (a free point on or off the segment or a point tied to another object). Kids always come in before and after school to perfect their figure or its animation. This project is a motivator for almost every single child!

Prerequisite Mathematical Terms and Concepts

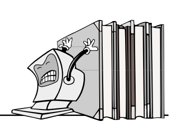


- *point, segment, polygon, polygon interior, vertex*
- names and properties of quadrilaterals, right angles and straight angles, sum of measures of central angles of a circle

Recommended Sketchpad Proficiency



- Basic knowledge of the freehand tools.
- Use of the Symmetry and Transformations Quick Reference Guide, found at the end of this unit, is suggested.
- Doing the Quadrilaterals unit prior to this one would be beneficial; otherwise, you may want to use specific sketches from that unit as student questions arise.



Essential Vocabulary

Angle of rotation—any angle of n degrees ($0 < n \leq 360$) through which a figure is rotated about a center point

Center of rotation—a point about which a figure can be rotated

Line of symmetry—a line or segment that separates a figure into two halves that are mirror images of each other

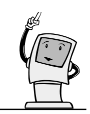
Midsegment—a segment connecting the midpoints of two sides of a polygon

Reflection symmetry—symmetry in which a line can be drawn that separates the given figure into two halves that are mirror images of each other

Rotation symmetry—symmetry in which a figure can be rotated about a center point by an angle of rotation so that it lies exactly atop its original position

Symmetry—balance; correspondence of parts of a figure on opposite sides of a point, line, or plane

Instructions for Teacher Demonstrations



In each unit, we have included sketches and guidelines for teacher demonstrations designed as introductions by the teacher in whole-class presentations before students attempt the various activities independently. They include the introduction of relevant mathematical vocabulary and concepts as well as appropriate Sketchpad skills. Often there are one or two activities that you will feel comfortable assigning to your students with very little guidance beforehand. In this unit, for example, **Mirrors and Reflection** works fine as an exploratory introduction to symmetry and requires very little technical expertise. On the other hand, one or more activities in each unit are designed to maximize guided discovery by the students. For these activities, a careful teacher demonstration will focus the students' attention on targeted mathematical concepts and prevent distractions due to lack of Sketchpad proficiency. In this unit, **Reflection and Rotation Symmetry** is designed to follow such a demonstration. Instructions are included later in these notes, along with answers for this guided discovery activity, for your convenience. Of course, you may want to use portions or adaptations of the demonstration before other activities; certainly, you will want to preview activities to be sure they are appropriate for the mathematical and Sketchpad skill levels of your students. In this Symmetry unit, at least a brief teacher demonstration is essential for success with most of the activities. Not only are the mathematical concepts new, but several Sketchpad skills that may be new to your students are also required in each activity. This situation calls for more preparation and practice than is necessary with most topics. Even with careful demonstrations, many students will forget one or more steps when they try for the first time, for example, to reflect a figure themselves at the computer. You need to show several examples and then have a student sit at the computer while another student gives verbal instructions at the close of the teacher demonstration.

We find it very valuable to solicit student suggestions during the demonstrations. In the rotation demonstration, for example, a student invariably first suggests trying an angle of rotation of 60° for the equilateral triangle. Errors like this stimulate thought, result in lively class discussion, and provide many opportunities for guided discovery.

You may find the Symmetry and Transformations Quick Reference Guide, found at the end of this unit, helpful in preparing for the symmetry demonstrations.

Reflection and Rotation Symmetry Activity

Exploring Reflection Symmetry

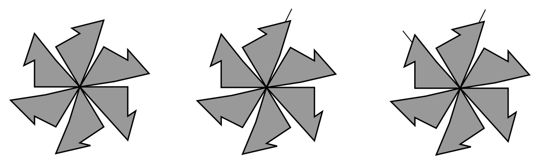
1. Discuss *symmetry*, *line of symmetry*, and *reflection symmetry*. Demonstrate and discuss the following Sketchpad steps as you show Symmetry Demo 1 (Mac) or SymDemo1.gsp (Windows).
   - Mark a segment as a mirror.
   - Select a figure using a selection marquee.
   - Reflect the figure over the mirror. (Discuss with students what they think the resulting figure will look like before reflecting.)
   - Drag parts of each design and discuss the effects of the changes.
2. Discuss testing a segment as a line of symmetry for a given figure. Demonstrate and discuss the following Sketchpad steps, using Symmetry Demo 2 (Mac) or SymDemo2.gsp (Windows).
   - Mark the dashed segment as a mirror.
   - Construct the polygon interior on one side of the segment.
   - Select the polygon interior and reflect it over the mirror.
   - Drag a vertex to show that the segment is a line of symmetry when the trapezoid is isosceles.
3. Using Symmetry Demo 3 (Mac) or SymDemo3.gsp (Windows), demonstrate constructing your own segments or lines to test figures for reflection symmetry.
   - Construct the midpoint of any side of the triangle and the segment connecting that midpoint to the opposite vertex.
   - Select the segment you have constructed and test your segment as a line of symmetry for the triangle.
   - Drag parts of the triangle and discuss the effects of the changes. Compare the results with those in Symmetry Demo 2. (Be sure students observe that the segment is a line of symmetry because the triangle remains equilateral, unlike the arbitrary trapezoid in Symmetry Demo 2, which must be manipulated to be isosceles.)

4. Demonstrate creating an original figure having a line of symmetry.
    - Open a new sketch and, using the Segment tool, draw a segment on the screen.
    - Use the Segment tool to draw a design on one side of the original segment.
    - Mark the original segment as a mirror.
    - Using a selection marquee, select the design you drew.
    - Reflect the design over the mirror.
    - Drag parts of the design to show that any changes made on one side of the mirror move in the same way on the other side of the mirror.

Exploring Rotation Symmetry

1. Discuss *symmetry, angle of rotation, center of rotation,* and *rotation symmetry.* Demonstrate and discuss testing a figure for rotation symmetry as you show Symmetry Demo 4 and Symmetry Demo 5 (Mac) or SymDemo4.gsp and SymDemo5.gsp (Windows).
    - In Symmetry Demo 4, select the shaded polygon interior, then rotate.
    - In Symmetry Demo 5, select an entire figure using a selection marquee when the figure is not shaded, then rotate.
    - Test a figure for a particular angle of rotation (e.g., 30° as test angle).
2. Demonstrate and discuss testing a figure for rotation symmetry. Demonstrate the following Sketchpad skills as you show Symmetry Demo 6 (Mac) or SymDemo6.gsp (Windows).
    - Mark the point as a center.
    - Select a figure to rotate, using both methods (as in Symmetry Demo 4 and Symmetry Demo 5).
    - Rotate a figure by a given angle (show rotations of 90° and 180°).
    - Find the angle(s) of rotation for a figure that makes the figure lie exactly atop its original position (take student suggestions until they find 120° and 240°).

Tip: We have found it helpful to place a tick mark on one edge of the figure so that when the figure is rotated by an angle that makes it fall exactly atop its original position, it is obvious to the student that the figure has moved. See below. Of course, you must select the tick mark before rotating the figure.



3. Demonstrate and discuss creating an original figure having rotation symmetry.
    - Open a new sketch and design a small, irregular polygon using the Segment tool.
    - Point out that a user may rotate a figure either by selecting only segments and points or by constructing the polygon interior and rotating.
    - Mark a point as a center (one of the lower vertices of the polygon often works well) and rotate the figure.
4. Review the sequence for rotating a figure.
    - Select the polygon (remind students of the tick mark suggestion).
    - Go to the **Transform** menu, choose **Rotate**, and make sure the **By Fixed Angle** option is chosen.
    - Type in the desired angle of rotation.
    - Repeat the rotation by returning to the **Transform** menu as many times as needed.
5. Demonstrate writing a student name on a sketch, using the Text tool.

You'll find examples of student sketches in the folders Student Reflection Sketches and Student Rotation Sketches (Mac) or Student_Ref and Student_Rot (Windows). These folders are in the Symmetry folder (Mac and Windows). You may want to show students these sketches to give them some ideas for their own sketches.

Writing Prompts

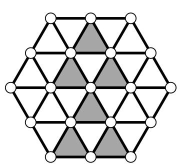
Ask students to choose a topic below and write a short paper, including sketches created with Sketchpad.

- Symmetry in Nature
- Symmetry in Quadrilaterals
- Symmetry in Regular Polygons
- Questions I Have About Symmetry

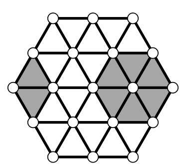
Examples of Student Work

Symmetry Project: Symmetric Patterns in a Hexagon

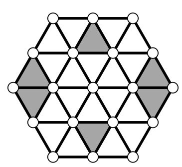
2a.



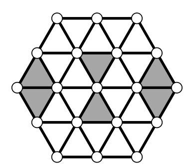
b.



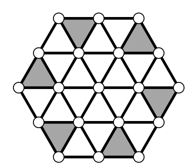
c.



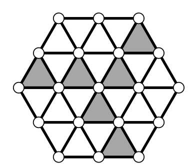
d.



e.

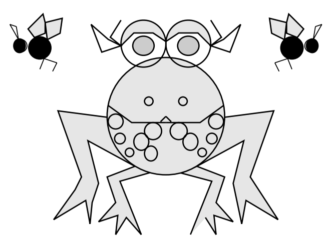


f.



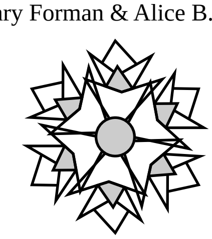
Symmetry Project: Make Me Symmetric

BY ALEXIS MCLEAN



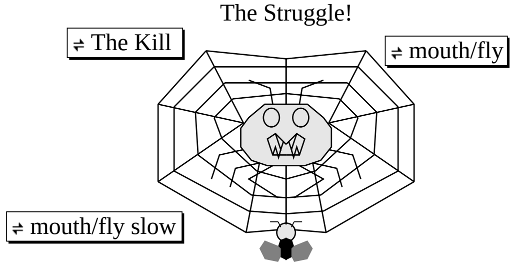
Ribbit Frog
This figure has one line of symmetry.

ROTATING ROSES
Mary Forman & Alice B. Cox



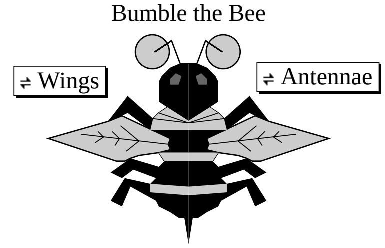
This figure has angles of rotation of 60°, 120°, 180°, 240°, and 300°.

The Struggle!



This figure has one line of symmetry.

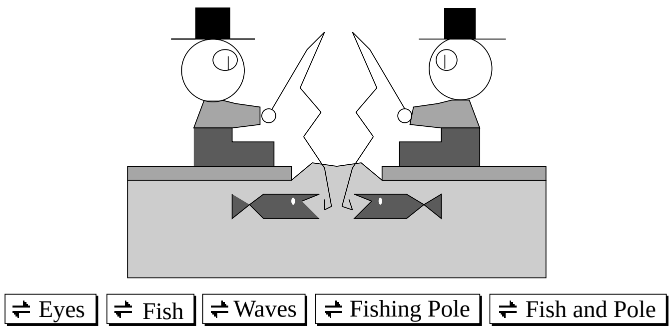
Bumble the Bee



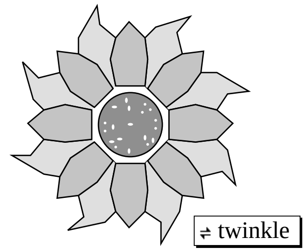
This figure has 1 line of symmetry.

Let's Boogie!

The Fishermen

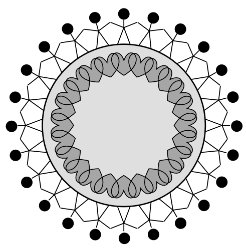


Sunflower



This figure has angles of rotation of 45, 90, 135, 180, 225, 270, & 315 degrees.

Love Your Planet



This figure has 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, 144, 156, 168, 180, 192, 204, 216, 228, 240, 252, 264, 276, 288, 300, 312, 324, 336, and 348 degree rotational symmetry.

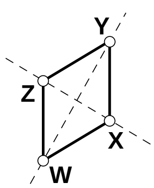
Kaleidoscope

THIS FIGURE HAS 90, 180, 270 DEGREE ROTATIONAL SYMMETRY.

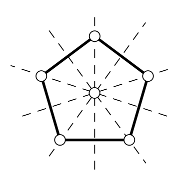
⇌ Rotate

Answers for Reflection and Rotation Symmetry Activity

4. a. no b. none c. trapezoid
   d. yes e. 2 f. rhombus
   g.



6. a. 5 b.



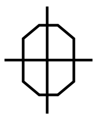
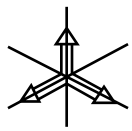
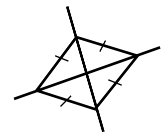
8. a. yes b. 120°, 240°, 360° c. yes
   d. 180°, 360°

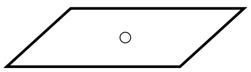
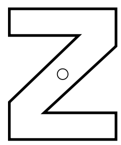
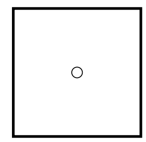
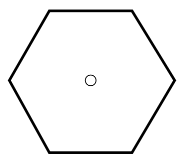
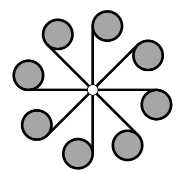
10. a. yes b. 60°, 120°, 180°, 240°, 300°, 360° c. yes
    d. 90°, 180°, 270°, 360°

12. a. yes b. 2 c. yes
    d. 180°, 360° e. no f. none
    g. yes h. 180°, 360°

Answers for Symmetry Wrap-Up

1. a. no b. yes c. no
   d. no e. yes f. yes

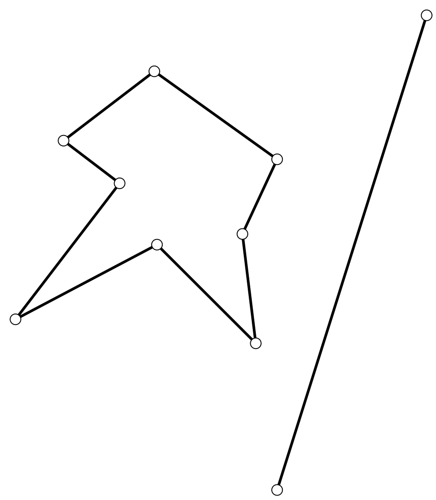
2. a.

   b.

   c.


3. a. no, yes

   b. no, yes

   c. yes, yes

   d. no, no
   e. no, yes

   f. yes, yes


4. a. Answers will vary. Possible answer: square.
   b. Answers will vary. Possible answer: parallelogram.
   c. Answers will vary. Possible answer: regular polygon with five or more sides.

Mirrors and Reflection

1. Open a new sketch.
2. Construct a segment.
3. Mark the segment as a mirror.
4. Construct a polygon on one side of the segment (see example).



5. Using the Selection Arrow tool, drag a selection marquee around the polygon to select it.
6. Reflect the polygon.
7. Drag any vertex of the polygon. Describe what happens.

____

____

____

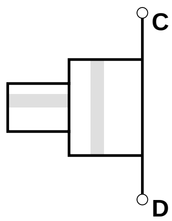
8. Drag the mirror. Describe what happens.

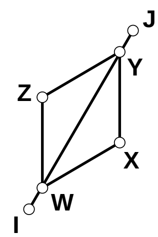
____

____

____

Reflection and Rotation Symmetry





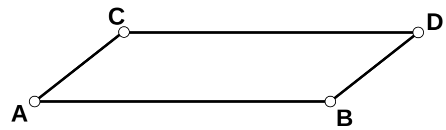
Use Sketchpad to explore reflection and rotation symmetry.

1. Open Symmetry Exp 1 (Mac) or Sym_Exp1.gsp (Windows).
2. Follow the instructions in the sketch.
3. Open Symmetry Exp 2 (Mac) or Sym_Exp2.gsp (Windows).
4. Follow the instructions in the sketch and answer these questions.
   a. Is the dashed segment **GH** a line of symmetry for quadrilateral **ADCB**? _________
   b. How many lines of symmetry do you think quadrilateral **ADCB** has? ___________
   c. What kind of quadrilateral is **ADCB**? ____________________________________
   You can measure the angles and sides if needed to identify the quadrilateral.
   d. Is the dashed segment **IJ** a line of symmetry for quadrilateral **WXYZ**? __________
   e. How many lines of symmetry do you think quadrilateral **WXYZ** has? ___________
   f. What kind of quadrilateral is **WXYZ**? ____________________________________
   You can measure the angles and sides if needed to identify the quadrilateral.
   g. Sketch quadrilateral **WXYZ** and show all its lines of symmetry here.

5. Open Symmetry Exp 3 (Mac) or Sym_Exp3.gsp (Windows).
6. Follow the instructions in the sketch and answer these questions.
   a. How many lines of symmetry does the regular pentagon have? _______________
   b. Sketch the regular pentagon and show all its lines of symmetry here.

7. Open Symmetry Exp 4 (Mac) or Sym_Exp4.gsp (Windows).
8. Follow the instructions in the sketch and answer these questions.
   a. Does figure E have rotation symmetry? ______________________________
   b. If so, list the angle(s) of rotation that make the figure lie atop its original position. ______________________________
   c. Does figure F have rotation symmetry? ______________________________
   d. If so, list the angle(s) of rotation that make the figure lie atop its original position. ______________________________
9. Open Symmetry Exp 5 (Mac) or Sym_Exp5.gsp (Windows).
10. Follow the instructions in the sketch and answer these questions.
    a. Does the regular hexagon in figure G have rotation symmetry? ____________
    b. If so, list the angle(s) of rotation that make the figure lie atop its original position. ______________________________
    c. Does figure H have rotation symmetry? ______________________________
    d. If so, list the angle(s) of rotation that make the figure lie atop its original position. ______________________________
11. Open Symmetry Exp 6 (Mac) or Sym_Exp6.gsp (Windows).
12. Follow the instructions in the sketch and answer these questions.
    a. Does figure I have reflection symmetry? ______________________________
    b. If so, how many lines of symmetry does it have? ____________________
    c. Does figure I have rotation symmetry? ______________________________
    d. If so, list the angle(s) of rotation that make the figure lie atop its original position. ______________________________
    e. Does figure J have reflection symmetry? ______________________________
    f. If so, how many lines of symmetry does it have? ____________________
    g. Does figure J have rotation symmetry? ______________________________
    h. If so, list the angle(s) of rotation that make the figure lie atop its original position. ______________________________

Diagonals in Parallelograms



1. Open Parallelogram Sym Diagonals (Mac) or ParaDiag.gsp (Windows).
2. Double click on the Show Diagonal **AD** button in the sketch to show the diagonal between points **A** and **D** and the polygon interior for Δ**ADC**.
3. Test diagonal **AD** as a line of symmetry for parallelogram **ABDC** by marking $\overline{\textbf{AD}}$ as a mirror, then reflecting the polygon interior of Δ**ADC** over the mirror. (See the Symmetry and Transformations Quick Reference Guide, if needed, for help.)
4. Drag your parallelogram into different types of parallelograms. Make sure to drag parallelogram **ABDC** so that it looks like a rectangle, a rhombus, and a square by dragging vertex **A**, **B**, **C**, or **D**.
5. Describe parallelogram **ABDC** when diagonal **AD** is a line of symmetry.

____

____

6. Double click on the Hide Diagonal **AD** and Interiors button in your sketch to hide $\overline{\textbf{AD}}$ and the polygon interiors.
7. Double click on the Show Diagonal **CB** button in the sketch.
8. Test diagonal **CB** as a line of symmetry for parallelogram **ABDC** by marking $\overline{\textbf{CB}}$ as a mirror, then reflecting the polygon interior of Δ**ABC** over the mirror.
9. Drag your parallelogram so that it looks like different types of parallelograms. Make sure to drag parallelogram **ABDC** into a rectangle, a rhombus, and a square by dragging vertex **A**, **B**, **C**, or **D**.
10. Describe parallelogram **ABDC** when diagonal **CB** is a line of symmetry.

____

____

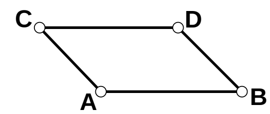
11. What conclusion(s) can you draw about the diagonals of a parallelogram as lines of symmetry?

____

____

____

Midsegments in Parallelograms



1. Open Parallelogram Sym Midsegments (Mac) or ParMdseg.gsp (Windows).
2. Double click on the Show Midsegment **FE** button to show the midsegment between midpoints **F** and **E** and the polygon interior for parallelogram **FEDC**.
3. Test midsegment **FE** as a line of symmetry for parallelogram **ABDC** by marking $\overline{FE}$ as a mirror, then reflecting the polygon interior of parallelogram **FEDC** over the mirror. (See the Symmetry and Transformations Quick Reference Guide, if needed, for help.)
4. Drag your parallelogram into different types of parallelograms. Make sure to drag parallelogram **ABDC** so that it looks like a rectangle, a rhombus, and a square by dragging vertex **A**, **B**, **C**, or **D**.
5. Describe parallelogram **ABDC** when midsegment **FE** is a line of symmetry.

____

____

6. Double click on the Hide **FE** and Interiors button in your sketch to hide midsegment **FE** and the polygon interiors.
7. Double click on the Show Midsegment **GH** button to show the midsegment between midpoints **G** and **H**.
8. Test midsegment **GH** as a line of symmetry for parallelogram **ABDC** by marking $\overline{GH}$ as a mirror, then reflecting the polygon interior of parallelogram **AGHC** over the mirror.
9. Drag your parallelogram so that it looks like different types of parallelograms. Make sure to drag parallelogram **ABDC** into a rectangle, a rhombus, and a square by dragging vertex **A**, **B**, C, or **D**.
10. Describe parallelogram **ABDC** when midsegment **GH** is a line of symmetry.

____

____

11. What conclusion(s) can you draw about the midsegments of a parallelogram as lines of symmetry?

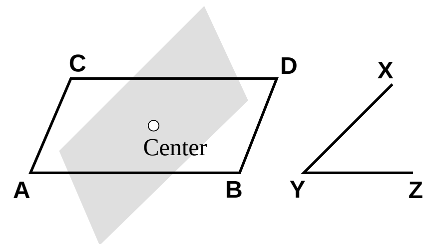
____

____

____

Rotation Symmetry in Parallelograms

1. Open the sketch Parallelogram Center (Mac) or ParaCntr.gsp (Windows). The polygon interior of parallelogram **ABDC** has been constructed, and it can be rotated about the point Center. The angle of rotation can be changed by dragging point **X** in the angle **XYZ**.



2. Drag point **X** to rotate the polygon interior. Find all angles of rotation less than 360° for which the polygon interior lies exactly atop parallelogram **ABDC**.
3. Does the general parallelogram displayed in this sketch have rotation symmetry for the following angles of rotation?

   90° ______________ 180° ______________ 270° ______________

4. Drag parallelogram **ABDC** into a rectangle by dragging vertex **A**, **B**, **D**, or **C**. Does the rectangle displayed in this sketch have rotation symmetry for the following angles of rotation?

   90° ______________ 180° ______________ 270° ______________

5. Drag parallelogram **ABDC** into a rhombus by dragging vertex **A**, **B**, **D**, or **C**. Does the rhombus displayed in this sketch have rotation symmetry for the following angles of rotation?

   90° ______________ 180° ______________ 270° ______________

6. Drag parallelogram **ABDC** into a square by dragging vertex **A**, **B**, **D**, or **C**. Does the square displayed in this sketch have rotation symmetry for the following angles of rotation?

   90° ______________ 180° ______________ 270° ______________

7. What conjecture(s) can you make about rotation symmetry in parallelograms?

   ____

   ____

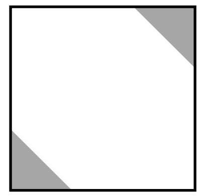
The S Files

1. Open the sketch The S Files (Mac) or S_Files.gsp (Windows).

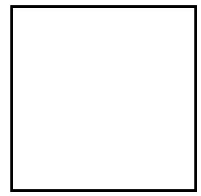
For questions 2–4 use Sketchpad to explore reflection and rotation symmetries in the figures in this sketch.

2. Compare the reflection and rotation symmetries in figure A to the symmetries of a square.

figure A

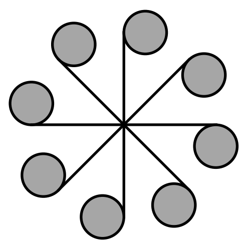


square

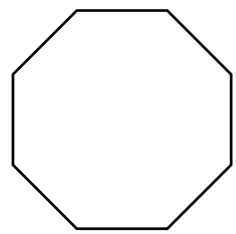


3. Compare the reflection and rotation symmetries in figure B to the symmetries of a regular octagon.

figure B

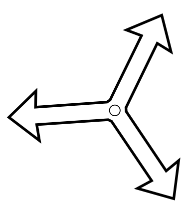


regular octagon

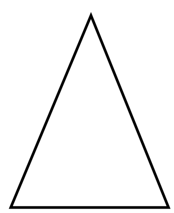


4. Compare the reflection and rotation symmetries in figure C to those of an isosceles triangle and to those of an equilateral triangle.

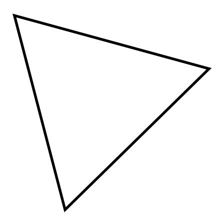
figure C



isosceles triangle

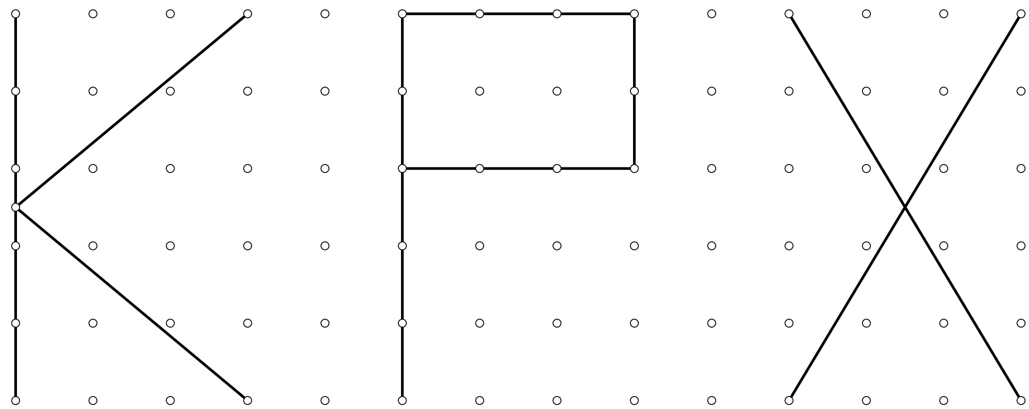


equilateral triangle



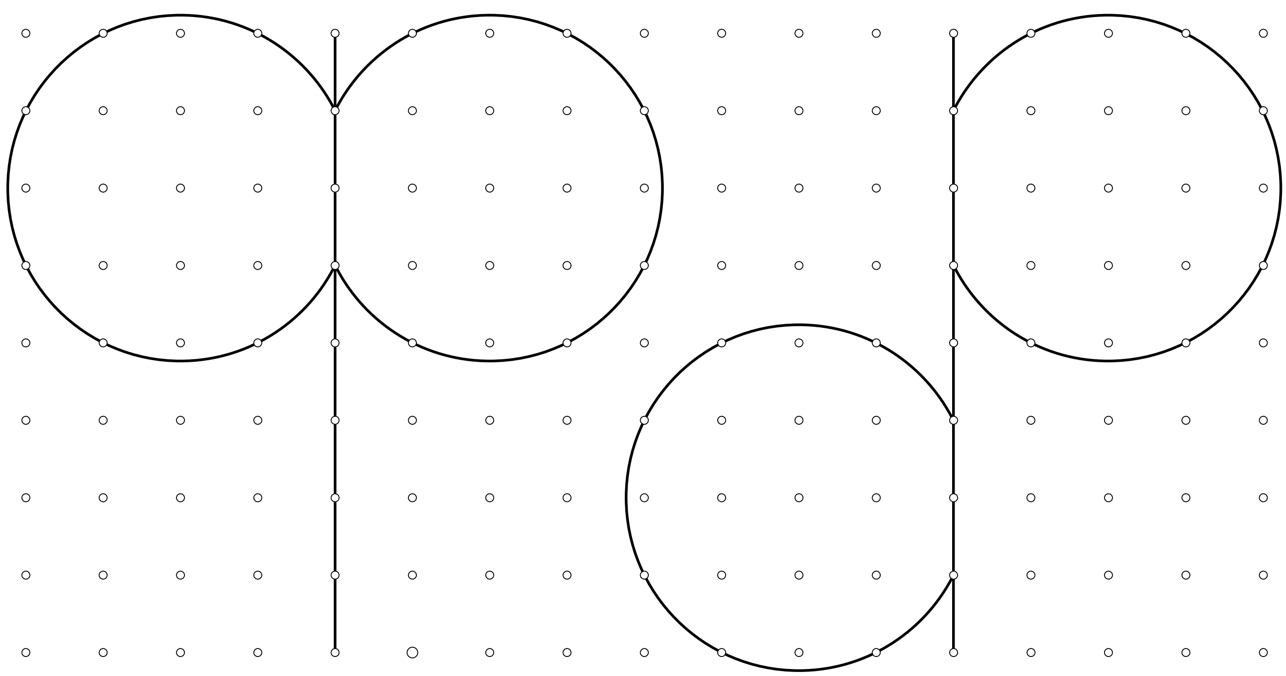
Alphabet Symmetry

Work with a partner or group to construct the 26 capital letters of our alphabet on the Sketchpad grid. Then classify them according to the reflection and rotation symmetries they have. Organize your results into a table.

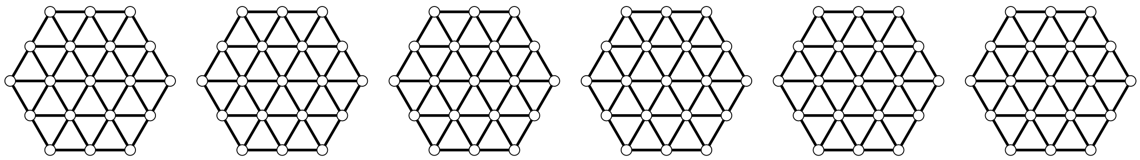


Extension: Design your own alphabet using Sketchpad so that every letter has reflection or rotation symmetry. You may want to change some of the traditional letters as little as possible (see the example), or you may want to completely design your own alphabet.

Example: For the letter P, you might choose to construct a design similar to one of those shown below.



Symmetric Patterns in a Hexagon



1. Open the sketch Hexagon Symmetry (Mac) or 6gon_Sym.gsp (Windows). You will see a sketch containing six congruent hexagons.

   To shade the inside of a figure using Sketchpad, you need to create the polygon interior of the polygon. To do this, use the Selection Arrow tool, hold down the Shift key, click on the vertices of the polygon in clockwise or counterclockwise order, go to the Construct menu, and choose Polygon Interior.

2. Shade one or more small triangles so that the completed design has the following reflection symmetry:
   a. Hexagon A has a vertical line of symmetry and no other.
   b. Hexagon B has a horizontal line of symmetry and no other.
   c. Hexagon C has both vertical and horizontal lines of symmetry.

   Shade in small triangles with a total area one-fourth that of the hexagonal region so that the completed design has the following rotation symmetry:
   d. Hexagon D has 180° rotation symmetry.
   e. Hexagon E has 60° rotation symmetry.
   f. Hexagon F has 120° rotation symmetry.

3. Test your answers to step 2 using reflections and rotations.

   To reflect an object, you must first mark a mirror. To do this, select a segment or a line, go to the **Transform** menu, and choose **Mark Mirror**. Then select the figure you wish to reflect, go to the **Transform** menu, and choose **Reflect**.

   To rotate an object, you must select a point using Sketchpad, go to the **Transform** menu, and choose **Mark Center**. Then select the figure you wish to rotate, go to the **Transform** menu, and choose **Rotate**. Enter an angle of rotation.

4. Put your name on your sketch and print out a copy to hand in.

Extension: Copy one of the original hexagons onto a new sketch. Shade triangles to create another design with both reflection and rotation symmetry. Write a statement on your sketch that tells what line(s) of symmetry it has and the angle(s) of rotation. Put your name on your sketch and print out a copy.

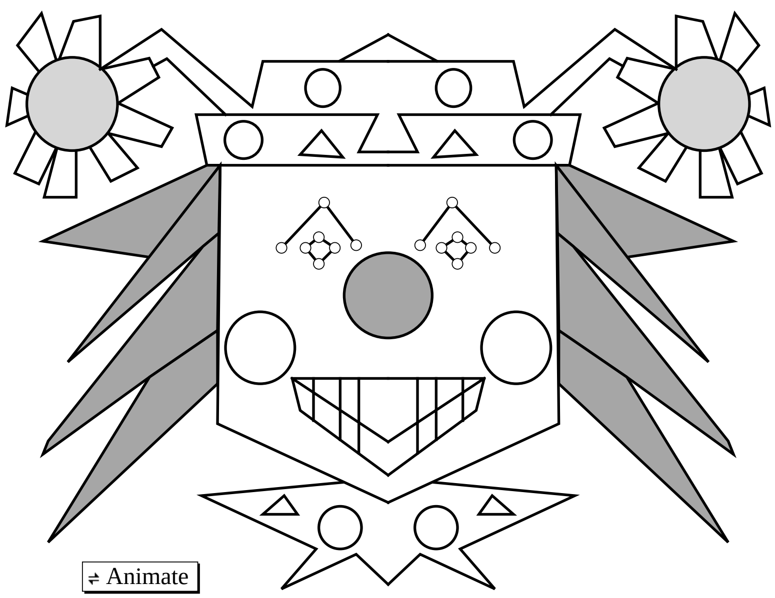
Make Me Symmetric

1. Use Sketchpad to create a figure with reflection symmetry. You must create a mirror and use reflection to give your figure at least one line of symmetry. Include a title, your name, and a statement describing the reflection symmetry of your figure.
2. Use Sketchpad to create a figure with rotation symmetry. Include a title, your name, and a statement describing the rotation symmetry of your figure.

Note: You can animate your figure by using the Sketchpad animation buttons. If you choose to include animation buttons, you will need to save the animated version on a disk.

THE CLOWN

SHARMAN RINGLAND & ALEXIS MCLEAN

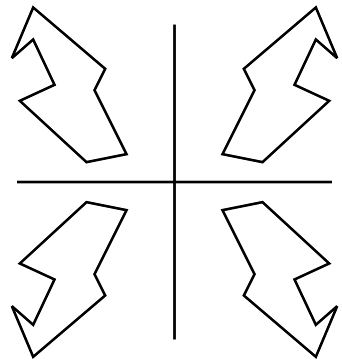


THIS FIGURE HAS ONE LINE OF SYMMETRY.

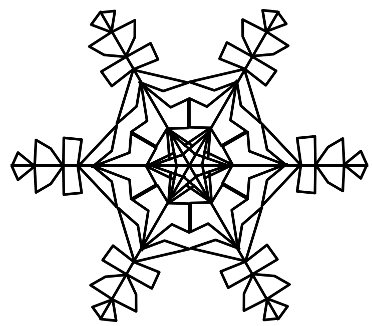
1. Find an example of reflection or rotation symmetry in the real world—a leaf, a tile pattern, the front of a building, etc. Open a new sketch and create a Sketchpad version of the object.

2. Create a figure with two lines of symmetry. Drag your figure to discover whether such a figure will *sometimes, always,* or *never* have rotation symmetry.

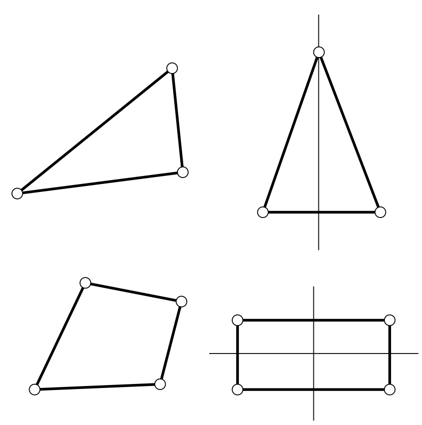
   Now try to create a figure with three lines of symmetry. Explore its rotation symmetry. Can you make a general statement about the relationship between the number of lines of reflection and the number of angles of rotation in these figures?



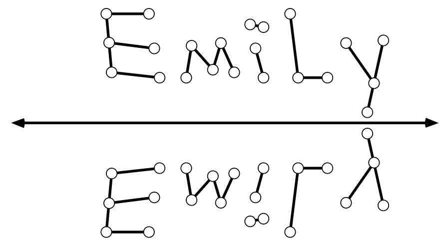
3. Read about snowflakes. Create your own version of a Sketchpad snowflake using reflections or rotations.



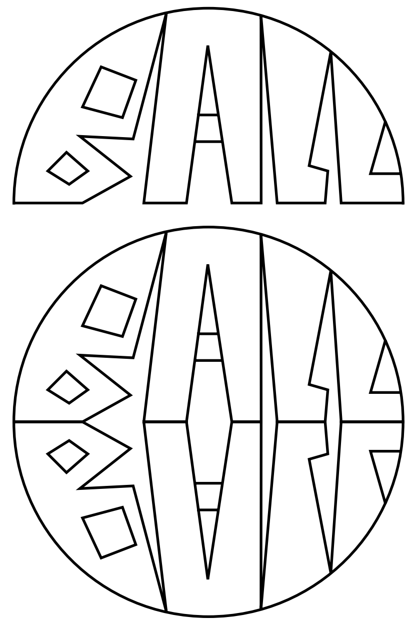
4. Use Sketchpad to create several polygons with different numbers of sides (first three, then four, five, and so on). Drag the vertices of each polygon so that it has reflection symmetry. After examining several such polygons, describe how the number of sides relates to the location of the line(s) of symmetry.



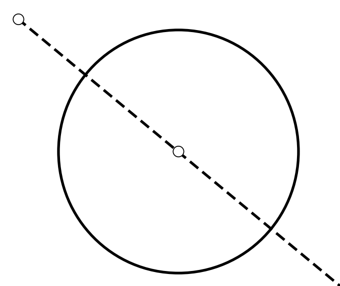
5. Open a new sketch and use the Segment tool to write your name. Reflect it over a line. Drag vertices to make it more interesting. Print a copy of your sketch.



Choose another word you think would make a nice visual image after it is reflected or rotated. Create your "picture word" using Sketchpad. Can you make your picture word show the meaning of the word as well?



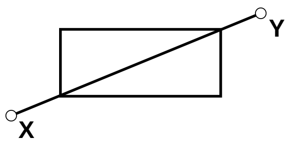
6. Investigate lines of symmetry in a circle. How many are there? What do all such lines of symmetry have in common? What can you say about rotation symmetry in a circle? Drag to try circles of different sizes. What conjectures can you propose?



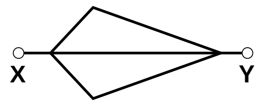
Reflection Symmetry

1. Is $\overline{XY}$ a line of symmetry? Write yes or no.

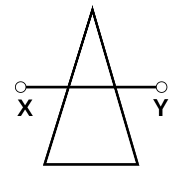
a. ________



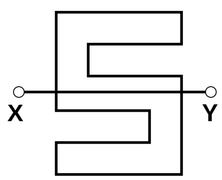
b. ________



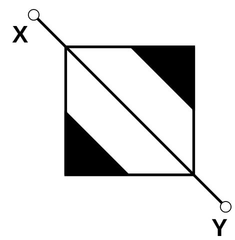
c. ________



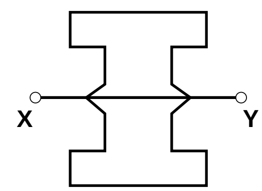
d. ________



e. ________

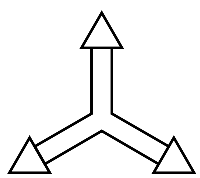


f. ________

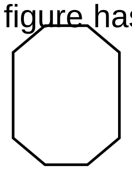


2. Draw all lines of symmetry for each figure. If the figure has no lines of symmetry, write “none.”

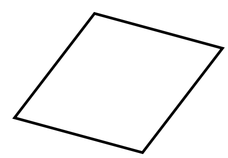
a. ________



b. ________



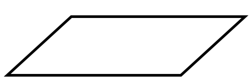
c. ________



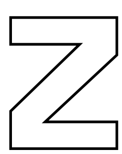
Rotation Symmetry

3. Does each figure have 90° rotation symmetry? Does each figure have 180° rotation symmetry? Write yes or no. If yes, mark the center of rotation.

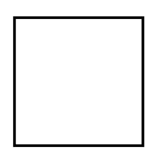
a. 90° ________
180° ________



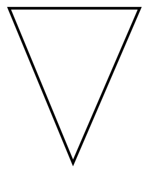
b. 90° ________
180° ________



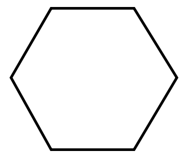
c. 90° ________
180° ________



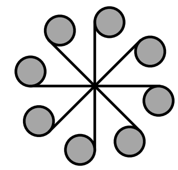
d. 90° __________
180° __________



e. 90° __________
180° __________



f. 90° __________
180° __________



4. Draw a figure that has the rotation symmetry listed.

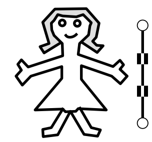
a. 90° and 180°

b. 180° only

c. an angle of rotation less than 90°

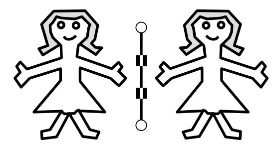
Symmetry and Transformations — Quick Reference Guide

To mark a segment or line as a mirror (use the Selection Arrow tool)



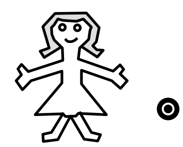
1. Click on the line or segment.
2. Go to the **Transform** menu and choose **Mark Mirror**.

To reflect a figure over a mirror



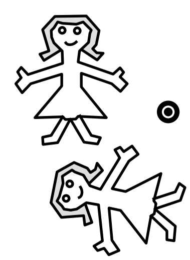
1. Mark a line or a segment as a mirror.
2. Select the figure.
3. Go to the **Transform** menu and choose **Reflect**.

To mark a point as a center (use the Selection Arrow tool)



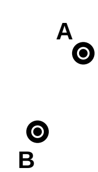
1. Click on the point.
2. Go to the **Transform** menu and choose **Mark Center**.

To rotate an object by a fixed angle



1. Mark a point as a center.
2. Select the figure.
3. Go to the **Transform** menu and choose **Rotate**.
4. Choose **By Fixed Angle**.
5. Enter the angle measure.
6. Click on **OK**.

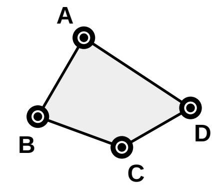
To construct a segment between two existing points (use the Selection Arrow tool)



1. Hold down the Shift key and click on the two points.
2. Go to the **Construct** menu and choose **Segment**.

To construct a polygon interior (use the Selection Arrow tool)

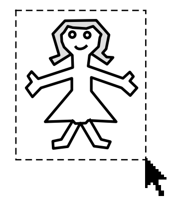
1. Hold down the Shift key. Click on the vertices of the polygon in order, either clockwise or counterclockwise.
2. Go to the **Construct** menu and choose **Polygon Interior**.



Select **A**, **B**, **C**, and **D**.

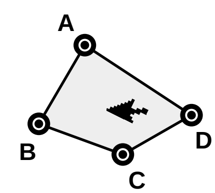
To select a figure using a selection marquee (use the Selection Arrow tool)

1. Click above and to the left of the figure. Drag the mouse down and to the right until the dashed rectangle surrounds the figure.
2. Release the mouse button. All objects inside the rectangle should be selected.



To select the polygon interior of a polygon (use the Selection Arrow tool)

1. Click on the shaded area of the polygon.

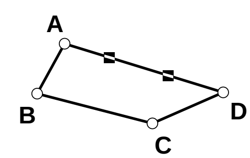


To write your name(s) on a sketch and then print (use the Text tool)

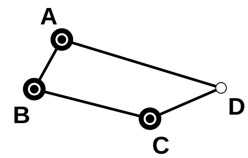
1. Drag in a blank area of your sketch to create a text box.
2. Type your name(s) in this box.
3. Go to the **File** menu and choose **Print**.

To measure segments or sides of polygons (use the Selection Arrow tool)

1. Click on the side or segment.
2. Go to the **Measure** menu and choose **Length**.
3. Drag the measure to a different location, if needed.



To measure angles (use the Selection Arrow tool)

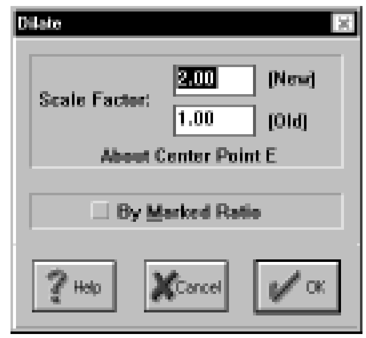


m∠ABC = 75°

1. Hold down the Shift key and click on the three points that define the angle. (Make sure you click on the vertex second—for ∠**B**, click on **A**, **B**, and then **C** or click on **C**, **B**, and then **A**.)
2. Go to the **Measure** menu and choose **Angle**.

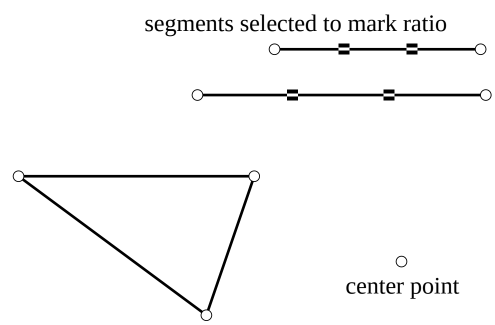
To dilate a figure by a fixed ratio

1. Select an existing point or use the Point tool to construct a point for your center.
2. Use the Selection Arrow tool to click on the point. Go to the **Transform** menu and choose **Mark Center**.
3. Use a selection marquee to select the figure. Go to the **Transform** menu and choose **Dilate**.
4. When the **Dilate** dialog box appears, enter new and old scale factors. Click on **OK**.

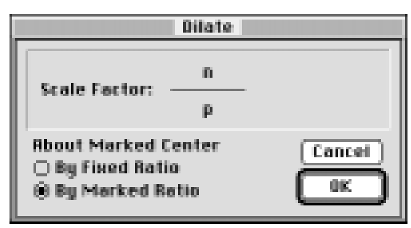


Dilate Dialog Box (Windows)

To dilate a figure by a marked ratio (use the Selection Arrow tool)



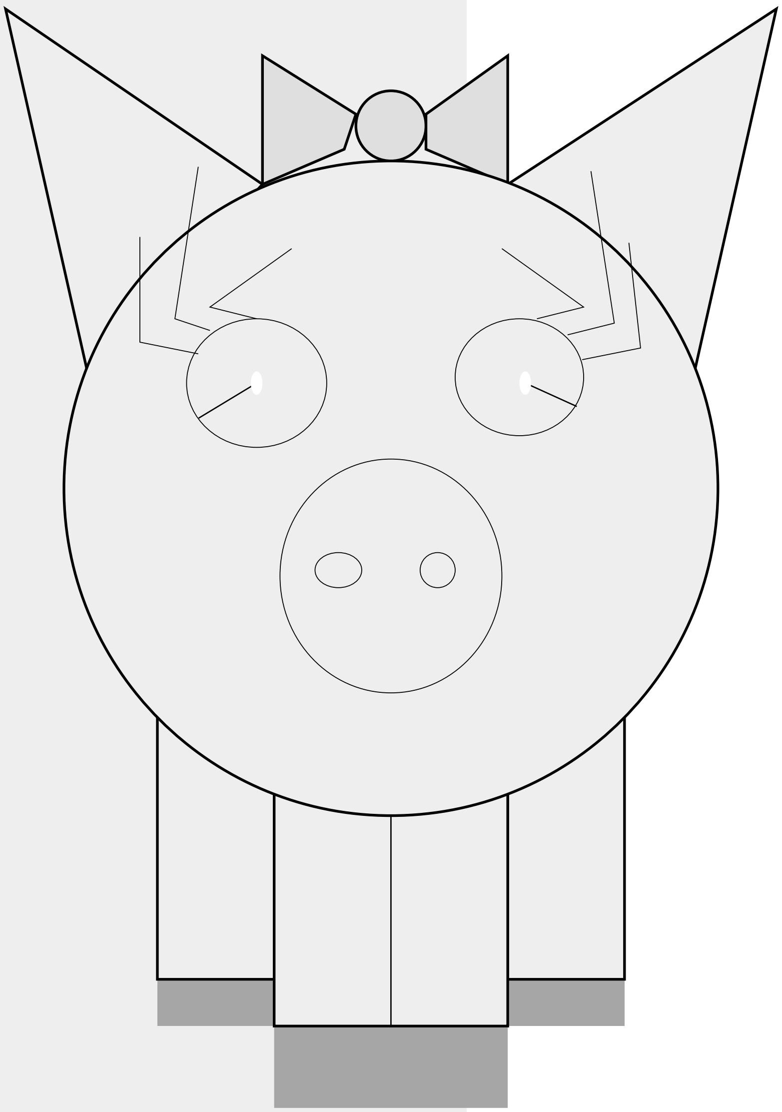
1. Click on a point to use as your center. Go to the **Transform** menu and choose **Mark Center**.
2. Construct two line segments in your sketch whose lengths show the ratio by which you wish to dilate. (Or select two line segments already in your sketch.)
3. Hold down the Shift key and click on both segments. Go to the **Transform** menu and choose **Mark Ratio**.
4. Use a selection marquee to select the figure. Go to the **Transform** menu and choose **Dilate**.
5. When the **Dilate** dialog box appears, choose **By Marked Ratio**. Click on **OK**.



Dilate Dialog Box (Macintosh)

Unit 6

TRANSFORMATIONS



Commentary

The topic of transformations has received varying degrees of emphasis in middle-school mathematics curricula in past decades. With increased use of computers in the classroom and workplace, however, the topic undoubtedly will receive increased attention at these grade levels. Investigating transformations fosters development of spatial reasoning and also builds a conceptual foundation for topics as varied as slope, trigonometric ratios, and fractals. As a significant bonus, middle-school kids love this topic!

Ideally, students doing the activities in this unit will have some Sketchpad proficiency. If not, the teacher demonstrations should receive special emphasis. Doing the activities in the Symmetry unit prior to this unit would be beneficial as well.

The activities **Exploring Translations** and **Exploring Rotations** are designed as introductory explorations for students who are unfamiliar with these concepts. The second part of each of these activities leads students through an alternate Sketchpad method for performing the transformations. You may want to omit these portions for some students until they are proficient in using the first method. It may also be appropriate to use the activity **Mirrors and Reflection** from the Symmetry unit with students who are being introduced to this topic for the first time. Of course, class discussion is needed as a follow-up to these explorations to clarify the definitions of the transformations for all your students.

During the teacher demonstration for **Exploring Transformations with Freehand Tools,** you will need to show the use of the Rotation tool with several figures. If students have trouble making the preimage and image match exactly, lead them to discover how to make this happen by moving the center of rotation. Also, it is best to let students suggest an alternate set of transformations for moving the same preimage to its image rather than just providing another solution for them. This teaching technique stimulates kids to visualize the results of various transformations in their minds and helps build confidence—even when several "undos" are needed before they are successful! We have found that this independent exploration helps students develop an accurate understanding of what happens to a figure during each kind of rigid transformation, an understanding that some fail to grasp when Sketchpad physically performs the transformation for them. Be sure students realize that a 90° rotation is in a counterclockwise direction, while a –90° rotation moves the figure clockwise. Student comments during these explorations are delightful: "No, no, we still gotta reflect—you got the fat leg on the skinny one!" and "That's not good enough. Move that center so the figures fit exactly!"

During the teacher demonstration for **Transformations on the Coordinate Grid,** you may want to show the students how to display coordinates of points with Sketchpad. However, we believe it is a good idea to have students at this level first name the coordinates themselves and then check their answers with Sketchpad.

For **Glide Reflections,** you may choose to omit showing your students translating a figure by a marked vector. For some groups you may want to show this at a later time, and for others, not at all. We have found it to be an alternative method that kids avoid if they do not understand it yet utilize quite efficiently when they do. As always, adjust your instruction to fit the needs of your students.

As you introduce dilations to your students in **Dilations,** it is important to point out the difference between this type of transformation and the rigid transformations (isometries) they have previously encountered. While most middle-school kids are familiar with photo enlargements and scale drawings or other practical applications of dilations, most likely they have not encountered the concept formally in their mathematics classes and often miss the distinction unless it is emphasized. For some groups of students, it may be appropriate to postpone the introduction of dilating a figure by a marked ratio until the first method has been completely mastered. Note that a third method, using the Dilation tool, is also available with Sketchpad. This activity brings "oohs" and "ahas" from students and stimulates many independent, self-initiated explorations.

Before students do **Similar Figures,** they often need a review of several Sketchpad skills and a teacher demonstration in addition to an introduction to corresponding sides and angles. We have found a dramatic improvement among students' recall of the important concepts related to similar figures since we started using this activity in our classes.

Your students will likely encounter a challenge with **Mission: Matched Pairs.** We suggest letting students work in pairs or teams. Another alternative is to require only 10 correct solutions instead of all 13. Despite their struggles, most of our students are quite engaged by this project and exhibit a real sense of pride when they are successful with it.

The Name Game is relatively simple, and the kids love to exhibit their results. As you can imagine, some produce quite elaborate creations!

Before students do **Triangle Buddies,** we find it important to demonstrate selecting the point at the intersection of the side of the triangle and the new parallel line and then constructing a new segment for each part of the original side. Point out to students that the measure of the length of a segment is the same as the distance between its two endpoints. Since measuring the distance between two points in this situation is a simpler maneuver with Sketchpad, you may want to encourage your students to use this method along with the **Calculate** feature from the **Measure** menu to find the ratio of the corresponding sides in the newly formed triangles.

For **Go, Team, Go,** you may want to expand the choices for your students to include an athletic court at a neighborhood recreation facility or nearby university. We have also varied this project to allow a scale drawing of any athletic court or field (in this case, checking the accuracy of student sketches becomes a challenge!). Kids enjoy sharing their sketches. If all students are assigned the same court, a good follow-up activity is to have students observe that all their scale drawings are dilations of the same preimage and, therefore, are all similar figures. You might have them find the scale factor between different groups' drawings. Working in pairs works well for this project.

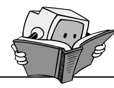
The results of **Golden Rectangle Survey** astound students every time we assign it. As long as the other three rectangles in their sketch are clearly *not* golden rectangles, more than 50% of the respondents to the survey always prefer their golden rectangle. You'll find many of your students quite eager to do more research and investigation on this topic. Be sure to discuss the questions listed in the "Think It Over" section of the project. Note that students will need to complete **Constructing a Golden Rectangle** (see the Constructions unit) as part of this project.

If you do not include dilations in your Transformations unit, you should omit questions 4 through 9 in the **Transformations Wrap-Up;** questions 1 through 3 relate to rigid transformations.

The project **Make Me Symmetric,** found in the Symmetry unit of this book, makes a very nice assessment activity at the completion of the Symmetry and Transformations units for middle-school students of all ages and abilities.

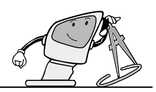
Several of the activities in the Constructions unit of this book involve transformations and/or similar polygons. Try doing **Constructing a Binary Tree Fractal, Constructing a Sierpinski Gasket Fractal, Constructing a Dragon Fractal, Constructing a Golden Rectangle,** or **Constructing a Pantograph** with your students. We have used each of these with great success!

Prerequisite Mathematical Terms and Concepts



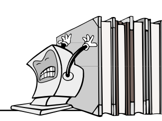
- *polygon interior, origin, coordinates of a point, horizontal, vertical, x-axis, y-axis, ratio, proportion*
- congruent figures, points on a coordinate plane, angles and angle measures, corresponding sides, corresponding angles, scale factor

Recommended Sketchpad Proficiency



- Basic knowledge of the freehand tools; some experience with Sketchpad is helpful.
- Use of the Symmetry and Transformations Quick Reference Guide (found at the end of the Symmetry unit) is suggested.
- Doing the Symmetry unit prior to this one would be beneficial; otherwise, you may want to use specific sketches from that unit as student questions arise.

Essential Vocabulary



Angle of rotation—any angle of n degrees ($0 < n < 360$) by which a figure is rotated around a point

Dilation—a transformation that shrinks or enlarges a figure

Glide reflection—a transformation that is a combination of a reflection and a translation

Image—a figure obtained through a transformation of a given figure

Isometry—movement of a figure so that the position of the figure changes but its size and shape do not

Line of reflection—line over which a preimage is reflected

Preimage—the original figure in a transformation

Reflection—a transformation that yields a figure that is a mirror image of the given figure

Rigid transformation (isometry)—movement of a figure so that the position of the figure changes but its size and shape do not; see also *reflection, rotation, translation*

Rotation—a transformation that rotates a figure a certain number of degrees about a point

Scale factor—the ratio of corresponding sides in two similar figures; given in Sketchpad as New : Old

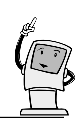
Similar polygons—two or more polygons that have all pairs of corresponding angles congruent and the same ratio for the lengths of all pairs of corresponding sides

Transformation—changing of a figure, as by a *reflection, rotation, translation,* or *dilation*

Translation—a transformation that moves all points in a figure by a given distance and in a given direction

Vector—a directed distance; can be used to define a translation

Instructions for Teacher Demonstrations



In each unit, we have included sketches and guidelines for teacher demonstrations designed as introductions by the teacher in whole-class presentations before students attempt the various activities independently. They include the introduction of relevant mathematical vocabulary and concepts as well as appropriate

Sketchpad skills. Often, there are one or several activities you will feel comfortable assigning to your students with very little guidance beforehand. In this unit, for example, the activities **Exploring Translations** and **Exploring Rotations** work fine as exploratory introductions to transformations and require very little technical expertise. On the other hand, one or more activities in each unit are designed to maximize guided discovery by the students. For these activities, a careful teacher demonstration will focus students' attention on targeted mathematical concepts and prevent distractions due to lack of Sketchpad proficiency. In this unit, the activities **Exploring Transformations with Freehand Tools, Transformations on the Coordinate Grid, Dilations,** and **Similar Figures** are designed to follow such a demonstration. The instructions are given below. You may want to use portions or adaptations of the demonstrations before doing other activities; certainly, you will want to preview activities to be sure they are appropriate for the mathematical and Sketchpad skill levels of your students.

If you choose to have your students work through several of the activities in this unit, you may want to show them the following shortcuts for marking mirrors and centers:

- To mark a mirror, simply double click on the line or segment you wish to designate as the mirror.
- To mark a center, you can double click on the point you wish to designate as the center.

It has been our experience with middle-school students that it is best to postpone the use of shortcuts until the students fully understand the steps that each shortcut replaces.

You may find the Symmetry and Transformations Quick Reference Guide (at the end of the Symmetry unit) helpful in preparing for the transformations demonstrations.

Exploring Transformations with Freehand Tools

1. Discuss *transformations* and *translations.* Demonstrate the following Sketchpad skills as you show Trans Demo 1 (Mac) or TranDem1.gsp (Windows):
   - Select the figure and copy it.
   - Paste the image and change its color or shade.
   - Use the Selection Arrow tool to drag (translate) the image. Discuss with students the relationship of the image and the preimage.
2. Discuss *reflections* and *lines of reflection.* Demonstrate the following Sketchpad skills as you show Trans Demo 2 (Mac) or TranDem2.gsp (Windows):
   - Mark a segment as a mirror.
   - Select a figure, using a selection marquee.
   - Reflect the figure over the mirror. Before reflecting, discuss with the students what they think the resulting figure will look like.
   - Drag the mirror and discuss the effects.
3. Discuss *rotations.* Demonstrate the following Sketchpad skills as you show Trans Demo 3 (Mac) or TranDem3.gsp (Windows):
   - Construct a point in the sketch and mark it as a center. (Discuss with the students where they think the center should be to make the preimage match the rotated image.)
   - Rotate the figure, using the Rotation tool .
   - Drag the center and discuss the effects. (You will have to use the Selection Arrow tool to drag the center.)
4. Show how different transformations of a preimage may be used to create the same

image, using Trans Demo 4 (Mac) or TranDem4.gsp (Windows) (e.g., create a rotated image and then show this same result by using two reflections).

- Double click on the Show Center button.
- Mark the point as a center.
- Select the preimage and rotate by an angle of 60°.
- Change the shade or color to clearly identify the image.
- Double click on the Show Mirror 1 button.
- Mark the segment as a mirror.
- Select the preimage and reflect.
- Double click on the Show Mirror 2 button.
- Mark the new segment as a mirror.
- Select the reflected image and reflect.

Transformations on the Coordinate Grid

Rigid Transformations

1. Review *transformations, reflections,* and *lines of reflection.* Demonstrate the following Sketchpad skills as you show Trans Demo 5 (Mac) or TranDem5.gsp (Windows):
   - Mark one of the axes as a mirror.
   - Select a polygon interior and the vertices.
   - Reflect the polygon over the mirror. Before you do the reflection, discuss with students what they think the image will look like.
2. Discuss *rotations.* Demonstrate the following Sketchpad skills, using Trans Demo 5 (Mac) or TranDem5.gsp (Windows):
   - Select a point (one of the vertices of the polygon or the origin) and mark it as a center.
   - Select the polygon and the vertices.
   - Choose **Rotate** from the **Transform** menu, enter an angle measure (we suggest 90° or 180°), and rotate the polygon. Discuss with students what they think the image will look like before you do the rotation.
3. Discuss *translations,* especially notation, such as $\langle 3, 2 \rangle$. Demonstrate the following Sketchpad skills, using Trans Demo 5 (Mac) or TranDem5.gsp (Windows):
   - Select the vertices and the polygon interior.
   - Choose **Translate** from the **Transform** menu.
   - Enter appropriate horizontal and vertical measures (make sure **By Rectangular Vector** is chosen) and translate the polygon.

Dilations

1. Review *transformations* and *similar polygons.* Use Dilate Demo 1 (Mac) or Dilate1.gsp (Windows) to illustrate a pair of similar polygons.

   a. Pairs of corresponding sides.
   - Select a pair of corresponding sides. Note that order is important.
   - Choose **Ratio** from the **Measure** menu.
   - Repeat for each pair of corresponding sides to show that the ratio is the same for each pair.
   - Drag any part of any figure and discuss the effects.

   b. Pairs of corresponding angles.
   - Select the three points that define one of the interior angles in quadrilateral **BARN**, in clockwise or counterclockwise order.
   - Choose **Angle** from the **Measure** menu.
   - Measure the corresponding angle in quadrilateral **SILO**. Compare the measures of the corresponding angles.

- Repeat for each pair of corresponding angles.
- Drag any part of any figure and discuss the effects.

2. Define a *dilation* and explain how it is different from an isometry. Review *corresponding sides* and *corresponding angles.* Review *ratios*, especially the 1:1 ratio for equal measures. Demonstrate the following Sketchpad skills, using Dilate Demo 2 (Mac) or Dilate2.gsp (Windows):

   a. Dilate a figure using a scale factor.
   - Select point **S** and choose **Mark Center** from the **Transform** menu.
   - Using a selection marquee, select hexagon **TAYLOR** and choose **Dilate** from the **Transform** menu.
   - For the Scale Factor, enter 1 for New and 2 for Old.
   - Drag any part of any figure and discuss the effects.

   b. Find the ratio of two corresponding sides.
   - Select a pair of corresponding sides in the preimage and the image. Note that order is important.
   - Choose **Ratio** from the **Measure** menu.
   - Repeat for each pair of corresponding sides.
   - Drag any part of any figure and discuss the effects.

   c. Compare the measures of corresponding angles.
   - Select the three points that define one of the interior angles in hexagon **TAYLOR**, in clockwise or counterclockwise order.
   - Choose **Angle** from the **Measure** menu.
   - Repeat for the corresponding angle of the image.
   - Compare the angle measures.
   - Repeat for each pair of corresponding angles.
   - Drag any part of any figure and discuss the effects.

3. Dilate a figure by a marked ratio. Demonstrate the following Sketchpad skills, using Dilate Demo 3 (Mac) or Dilate3.gsp (Windows):
   - Select point **D** and choose **Mark Center** from the **Transform** menu.
   - Hold down the Shift key. Select $\overline{\mathbf{LM}}$ and then $\overline{\mathbf{JK}}$. Choose **Mark Ratio** from the **Transform** menu.
   - Using a selection marquee, select pentagon **MARGE** and choose **Dilate** from the **Transform** menu.
   - Choose **By Marked Ratio**.
   - Drag **K** in the sketch and discuss how the figure changes.
   - Discuss or demonstrate the ratio of corresponding sides and corresponding angles.
   - Drag any part of any figure and discuss the effects.

4. Dilate a figure using the Dilation tool. Demonstrate the following Sketchpad skills, using **Dilate Demo 4** (Mac) or **Dilate4.gsp** (Windows):
   - Select point **D** and choose **Mark Center** from the **Transform** menu.
   - Press and hold down the mouse button on the Selection Arrow tool. Drag to the right and choose the Dilation tool.
   - Select the entire figure **PAUL**, using a selection marquee, and drag to show the effect of using the Dilation tool.

Similar Figures

1. Discuss *polygons.* Review the meaning of *ratio.* Demonstrate the following Sketchpad skills, using Similar Demo 1 (Mac) or SimDemo1.gsp (Windows):
   a. Measure an angle.
      - Select the three points that define the angle, in clockwise or counterclockwise order.
      - Choose **Angle** from the **Measure** menu.

   b. Find the ratio of two segments.
      - Select the segments.
      - Choose **Ratio** from the **Measure** menu.

   c. Change the name of a measure (a name often needs shortening).
2. Discuss corresponding sides and corresponding angles in a pair of polygons. Using Similar Demo 1 (Mac) or SimDemo1.gsp (Windows), have students identify the following:
   - Each pair of corresponding sides in quadrilaterals **BARN** and **SILO**.
   - Each pair of corresponding angles in quadrilaterals **BARN** and **SILO**.

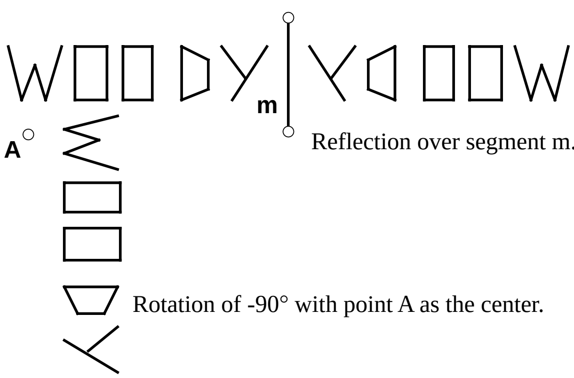
Writing Prompts

Ask students to choose a topic below and write a short paper, including sketches created with Sketchpad.

- Flip, Slide, and Turn—The Basics of Transformations
- Groovin' on the Grid (Transformations on the Coordinate Plane)
- Similarity Simplified
- Up, Down, and All Around—Transformations in the Real World
- If I Were a Transformation

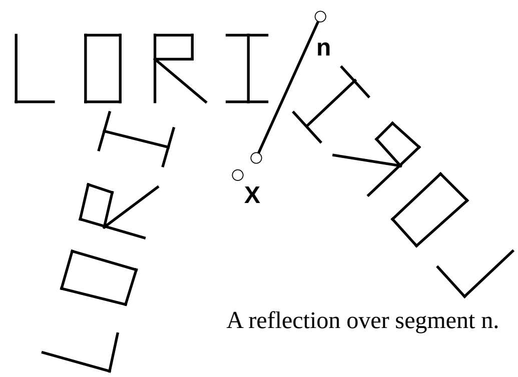
Examples of Student Work

The Name Game



Reflection over segment m.

Rotation of -90° with point A as the center.

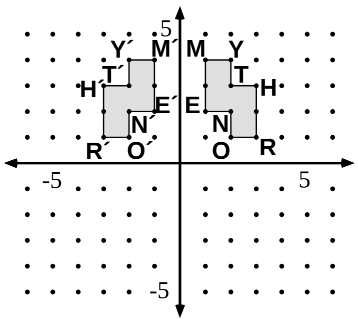


A reflection over segment n.

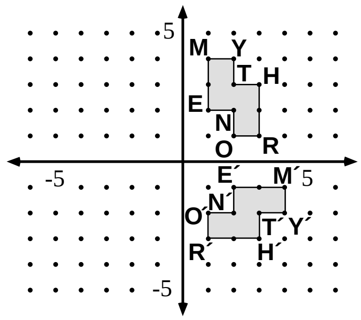
A rotation of 75° with X as the center.

Answers for Transformations on the Coordinate Grid Activity

2.



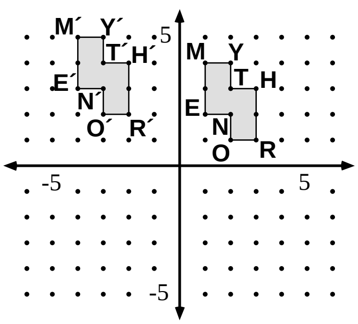
6.



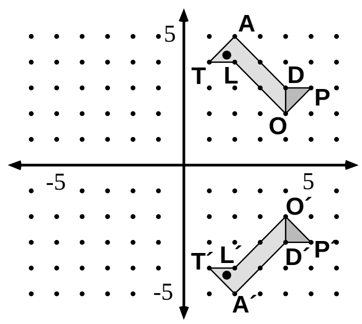
4. **M´** (–1, 4) **R´** (–3, 1)

8. **M´** (4, –1) **R´** (1, –3)

10.



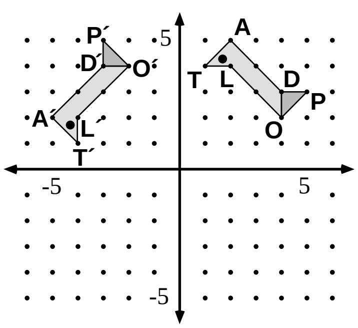
14.



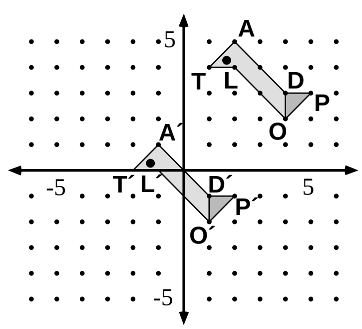
12. **M´** (–4, 5) **R´** (–2, 2)

16. **A´** (2, –5) **P´** (5, –3)

18.



22.



20. **A´** (–5, 2) **P´** (–3, 5)

24. **A´** (–1, 1) **P´** (2, –1)

Answers for Dilations Activity

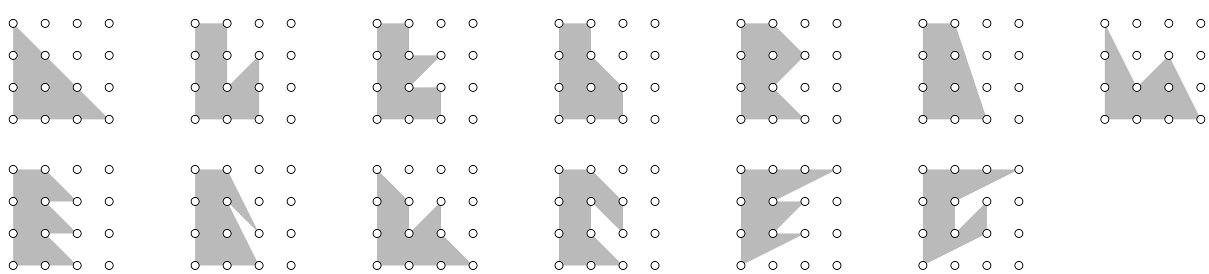
3. 0.75
4. Each side in the image is three-fourths as long as the corresponding side in the preimage. Alternative answer: The ratio of the measures of the lengths of each pair of corresponding sides is 0.75.
5. They are equal.
6. a. They are equal. b. 1.00, or 1 : 1

7. a. 1.25 b. 1.00, or 1 : 1
8. The ratio of the measures of the lengths of each pair of corresponding sides will be 0.67, or 2 : 3. The ratio of the measures of each pair of corresponding angles is 1.00, or 1 : 1.
12. Both ratios are 0.5.
13. yes
14. They are equal.
15. a. They are equal. b. 1.00, or 1 : 1
    c. Yes. The ratio of the measures of the lengths of every pair of corresponding sides is the same. The measures of every pair of corresponding angles are the same.
16. a. When $\overline{EF}$ is shorter than $\overline{GH}$, the ratio is less than 1.00 and the image is smaller than the preimage.
    b. When $\overline{EF}$ is longer than $\overline{GH}$, the ratio is greater than 1.00 and the image is larger than the preimage.
    c. When $\overline{EF}$ and $\overline{GH}$ have equal lengths, the ratio is 1.00 and the figures coincide.

Answers for Similar Figures Activity

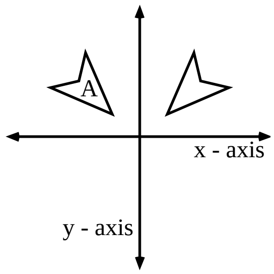
2. a. $\overline{TI}$, $\overline{IN}$, $\overline{NY}$, $\overline{YT}$, b. 2.0 (or 0.5)
   c. $\angle YTI$, $\angle TIN$, $\angle INY$, $\angle NYT$ d. They are equal.
4. **REDIPS**; $m\angle YWE \neq m\angle EDI$
6. **LAUGH**; $m\angle EIN \neq m\angle HGU$
8. a. 1.5 b. 1.5
   c. $\overline{FT}$ = 4 inches; $\frac{6}{x} = \frac{3}{2}$ d. Answers will vary.
   e. Same measure as 8d; corresponding angles of similar triangles are congruent.

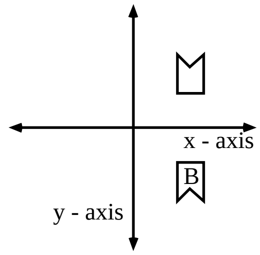
Solution for Mission: Matched Pairs Project

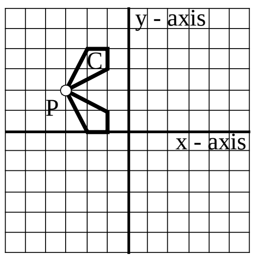


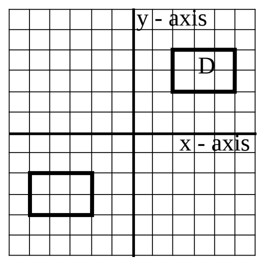
Answers for Transformations Wrap-Up

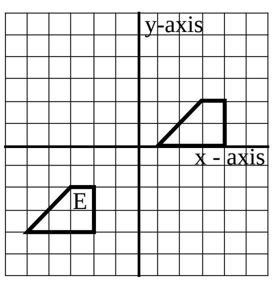
1. a. C, D b. B, E c. F
2. a. reflection b. rotation c. rotation
   d. rotation e. translation

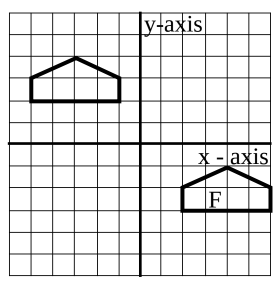
3. a.


b.


c.


d.


e.


f.


4. $\overline{\textbf{WO}}$

5. $\angle$**YTS**

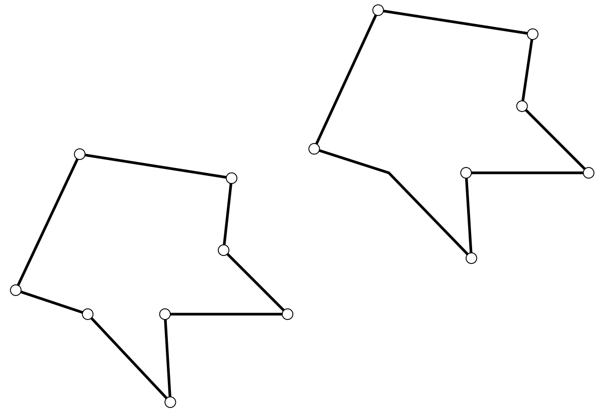
6. $\overline{\textbf{BI}}$

7. $\angle$**PEA** or $\angle$**AEP**

8. 1.2 units

9. 60°

Exploring Translations

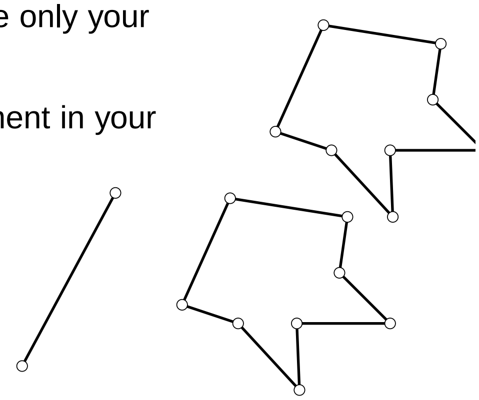


1. Open a new sketch.
2. Construct a polygon (use the Segment tool). This figure is your preimage.

Use Sketchpad to translate your polygon.

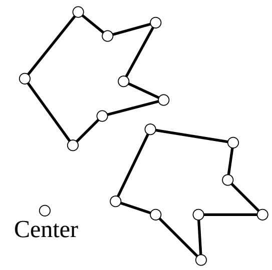
3. Select your polygon (use the Selection Arrow tool).
4. Go to the **Transform** menu and choose **Translate**.
5. Choose **By Rectangular Vector**.
6. Enter a Horizontal and a Vertical distance.
7. Click on **OK**. This new figure is your image.
8. Drag any vertex or side of your preimage. Observe the effects on the image.
9. Go to the **Edit** menu and choose **Undo** until you see only your original selected preimage.
10. Repeat steps 4, 5, 6, and 7, entering different values for the Horizontal and Vertical distances. You may want to use some negative values. Observe the results.
11. Drag your figure again and make observations.
12. Based on your observations, define *translation*. Include the relationship between sizes, shapes, and locations of the preimage and image.

Alternate Method for Translations



13. Go to the **Edit** menu and choose **Undo** until you see only your original preimage.
14. Using the Segment tool, construct a short segment in your sketch.
15. Hold down the Shift key, and click on the two endpoints of the segment. Make sure only the endpoints are selected, not the segment itself.
16. Go to the **Transform** menu and choose **Mark Vector**.
17. Select the sides and vertices of your polygon (use the Selection Arrow tool).
18. Go to the **Transform** menu and choose **Translate**.
19. Choose **By Marked Vector**.
20. Click on **OK**. You should see a new image appear.
21. Drag one of the endpoints of the vector segment. Observe the image as you drag either endpoint of the segment. How does this affect the size, shape, and position of the image?
22. Drag one of the vertices of the preimage. How does this affect the size, shape, and position of the image?
23. How is the effect of changing the vector different from the effect of changing the preimage?
24. Compare the two methods you have learned for translating a figure.

Exploring Rotations



1. Open a new sketch.
2. Construct a polygon (use the Segment tool). This figure is your preimage.

Use Sketchpad to rotate your polygon.

3. Construct a point near your polygon (use the Point tool).
4. While this point is selected, go to the **Transform** menu and choose **Mark Center**.
5. Select the sides and vertices of your polygon (use the Selection Arrow tool).
6. Go to the **Transform** menu and choose **Rotate**.
7. Choose **By Fixed Angle**.
8. Enter an angle measure.
9. Click on **OK**. This new figure is your image.
10. Drag any vertex or side of your preimage. Observe the effects on the image.
11. Go to the **Edit** menu and choose **Undo** until you see only your original preimage.
12. Repeat steps 5, 6, 7, 8, and 9, entering different values for the angle of rotation. You may want to use some negative values. Observe the results.
13. Drag your figure again and make observations.
14. Based on your observations, define *rotation*. Include the relationship between sizes, shapes, and locations of the preimage and image.

____

____

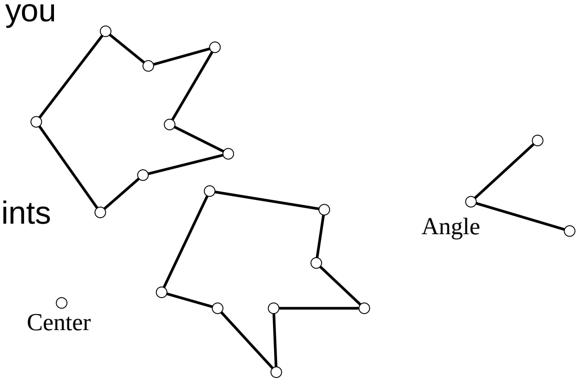
15. Drag the point you marked as your center of rotation. Make observations. Use your observations to define a *center of rotation.*

____

____

____

Alternate Method for Rotations



16. Go to the **Edit** menu and choose **Undo** until you see only your original preimage.
17. Construct an angle in your sketch (use the Segment tool ⟋).
18. Hold down the Shift key, and click on the points that define the angle, in clockwise or counterclockwise order. Make sure the vertex of the angle is the second point you choose (use the Selection Arrow tool ↖).
19. Go to the **Transform** menu and choose **Mark Angle**.
20. Select the sides and angles of your polygon (use the Selection Arrow tool ↖).
21. Go to the **Transform** menu and choose **Rotate**.
22. Choose **By Marked Angle**.
23. Click on **OK**. You should see a new image appear.
24. Drag one of the points defining your angle of rotation. Observe the image as you drag. How does this affect the size, shape, and position of the image?

    ____

    ____

25. Drag one of the vertices of the preimage. How does this affect the size, shape, and position of the image?

    ____

    ____

26. How is the effect of changing the angle of rotation different from the effect of changing the preimage?

    ____

    ____

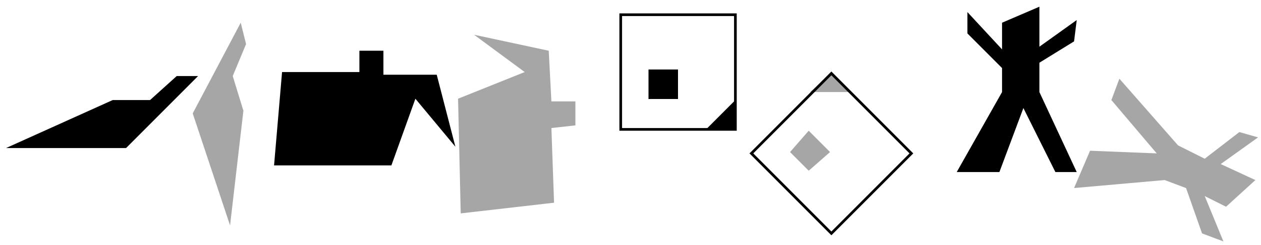
27. Compare the two methods you have learned for rotating a figure.

    ____

    ____

    ____

Exploring Transformations with Freehand Tools



Use the freehand tools with the **Transform** menu in Sketchpad to explore transformations.

1. Open the sketch Moves 1 (Mac) or Moves_1.gsp (Windows).
2. Use reflections, rotations, and/or translations to move the darker shaded figure on top of the other figure.
3. Open the sketch Moves 2 (Mac) or Moves_2.gsp (Windows).
4. Use reflections, rotations, and/or translations to move the darker shaded figure on top of the other figure.
5. Open the sketch Moves 3 (Mac) or Moves_3.gsp (Windows).
6. Use reflections, rotations, and/or translations to move the darker shaded figure on top of the other figure.
7. Open the sketch Moves 4 (Mac) or Moves_4.gsp (Windows).
8. Use reflections, rotations, and/or translations to move the darker shaded figure on top of the other figure. Use the space below to briefly describe the transformations you used to make this happen.

___

___

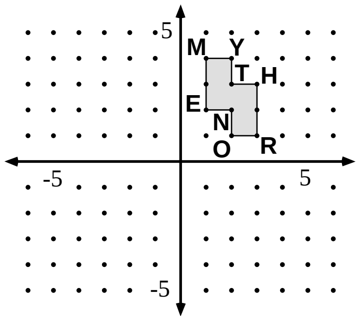
___

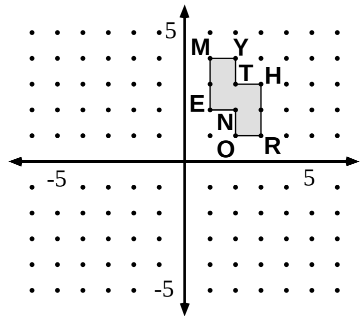
On Your Own: Reopen one of the sketches in this activity. Try to use a different transformation or set of transformations than you did the first time to move the darker shaded figure onto the lighter figure.

Transformations on the Coordinate Grid

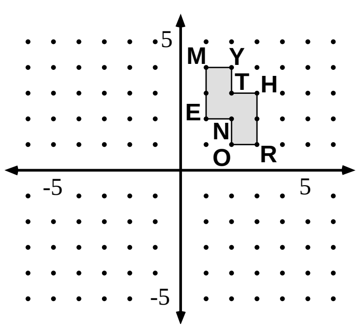
Use Sketchpad to explore transformations on the coordinate plane.

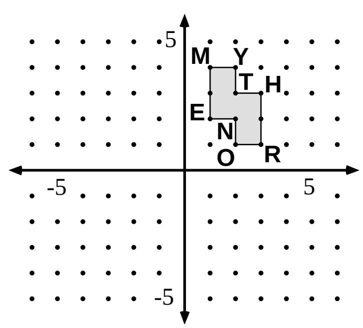
1. Open the sketch Grids 1 (Mac) or Grids_1.gsp (Windows).
2. Imagine the image of polygon **MYTHRONE** after it is reflected over the y-axis. On the grid at left below, sketch the image as you think it will appear.





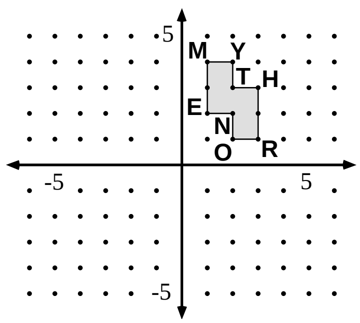
3. Reflect **MYTHRONE** over the y-axis.
   a. Mark the y-axis as a mirror.
   b. Select the vertices, sides, and polygon interior of **MYTHRONE**.
   c. Go to the **Transform** menu and choose **Reflect**.
4. On the grid at right above, sketch the image of **MYTHRONE** after reflection. Give the coordinates of **M´** and **R´**. **M´**(__________) **R´**(__________)
5. Go to the **Edit** menu and choose **Undo** until your reflection no longer appears.
6. Imagine the image of **MYTHRONE** after it is rotated 90° clockwise about the origin. On the grid at left below, sketch the image as you think it will appear.

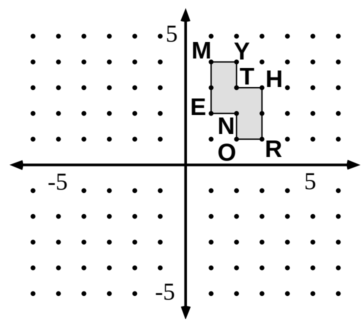




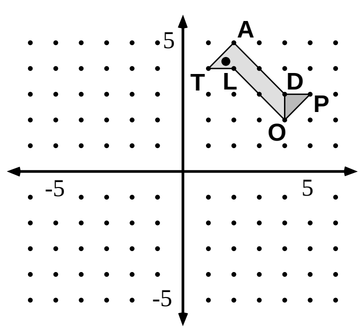
7. Rotate **MYTHRONE** –90° about the origin.
   a. Mark the origin as a center.
   b. Select the vertices, sides, and polygon interior of **MYTHRONE**.
   c. Go to the **Transform** menu and choose **Rotate**. Enter –90.

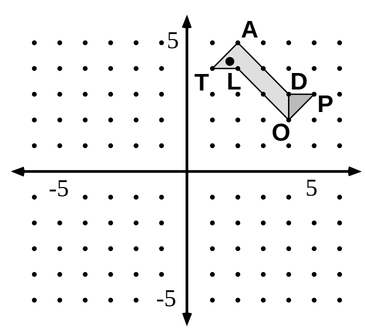
8. On the grid at right in step 6, sketch the image of **MYTHRONE** after the rotation. Give the coordinates of **M´** and **R´**. **M´**(__________) **R´**(__________)
9. Go to the **Edit** menu and choose **Undo** until your rotation no longer appears.
10. Imagine the image of **MYTHRONE** after it is translated by ⟨–5, 1⟩. (Remember that ⟨–5, 1⟩ means to move each point 5 units to the left and 1 unit up.) On the grid to the left below, sketch the image as you think it will appear.



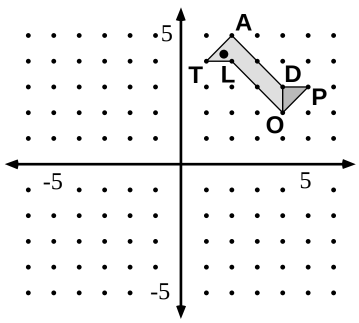


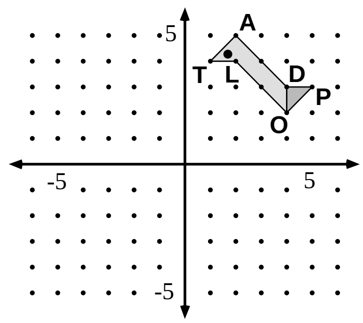
11. Translate **MYTHRONE** by ⟨–5, 1⟩.
    a. Select the vertices, sides, and polygon interior of **MYTHRONE**.
    b. Go to the **Transform** menu and choose **Translate**.
    c. Be sure **By Rectangular Vector** is chosen.
    d. Enter –5 as the Horizontal value and 1 as the Vertical value, and click on **OK**.
12. On the grid to the right above, sketch the image of **MYTHRONE** after the translation. Give the coordinates of **M´** and **R´**. **M´**(__________) **R´**(__________)
13. Open the sketch Grids 2 (Mac) or Grids_2.gsp (Windows).
14. Imagine the image of **TADPOL** after it is reflected over the x-axis. On the grid at left below, sketch the image as you think it will appear.



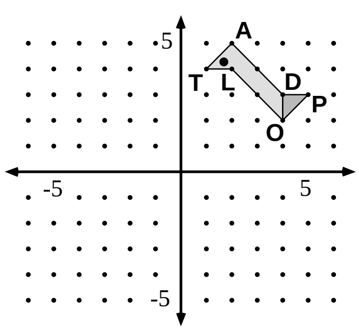


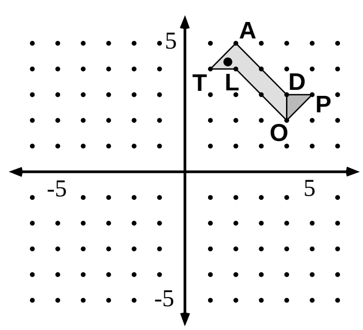
15. Reflect **TADPOL** over the x-axis.
    a. Mark the x-axis as a mirror.
    b. Select the vertices, sides, and polygon interiors of **TADPOL**.
    c. Chose **Reflect** from the **Transform** menu, and reflect the preimage.
16. On the grid at right in step 14, sketch the image of **TADPOL** after the reflection. Give the coordinates of **A´** and **P´**. **A´**(__________) **P´**(__________)
17. Go to the **Edit** menu and choose **Undo** until your reflection no longer appears.
18. Imagine the image of **TADPOL** after it is rotated 90° counterclockwise about the origin. On the grid at left below, sketch the image as you think it will appear.





19. Rotate **TADPOL** 90° about the origin.
    a. Mark the origin as a center.
    b. Select the vertices, sides, and polygon interiors of **TADPOL**.
    c. Choose **Rotate** from the **Transform** menu, enter 90, and click on **OK**.
20. On the grid at right above, sketch the image of **TADPOL** after the rotation. Give the coordinates of **A´** and **P´**. **A´**(__________) **P´**(__________)
21. Go to the **Edit** menu and choose **Undo** until your rotation no longer appears.
22. Imagine the image of **TADPOL** after it is translated by $\langle -3, -4 \rangle$. On the grid at left below, sketch the image as you think it will appear.





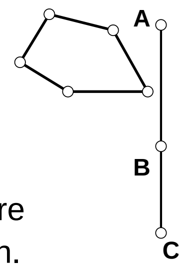
23. Translate **TADPOL** by $\langle -3, -4 \rangle$.
    a. Select the vertices, sides, and polygon interiors of **TADPOL**.
    b. Choose **Translate** from the **Transform** menu.
    c. Be sure **By Rectangular Vector** is chosen.
    d. Enter –3 as the Horizontal value and –4 as the Vertical value, and click on **OK**.
24. On the grid at right in step 22, sketch the image of **TADPOL** after the translation.

    Give the coordinates of **A´** and **P´**. **A´**(__________) **P´**(__________)

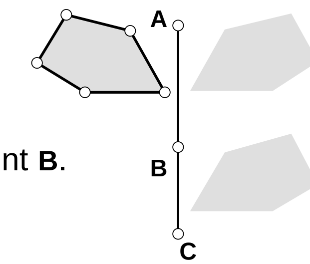
On Your Own: Reopen the sketch Grids 1 (Mac) or Grids_1.gsp (Windows). Perform a transformation somewhat different from any in this activity. Then perform a transformation on the image you just created. Print out the sketch. Now hide the first image and print out the sketch again. Compare second printouts with your partner to see whether each of you can guess the two transformations used by the other to create the final image. Turn in all printouts.

Glide Reflections

A *glide reflection* is a combination of two transformations. Use Sketchpad to explore glide reflections.



1. Open a new sketch.
2. Use the Segment tool to draw a polygon.
3. Construct the polygon interior.
4. Construct a segment and a point on the segment. The points are labeled **A**, **B**, and **C** in the diagram at right for easy identification.
5. Mark your segment as a mirror.



6. Select the polygon interior, go to the **Transform** menu, and choose **Reflect**.
7. Use the Selection Arrow tool to select point **A** and then point **B**.
8. Go to the **Transform** menu and choose **Mark Vector**.
9. Select the image (the reflected polygon). Go to the **Transform** menu, choose **Translate**, and choose **By Marked Vector**. Click on **OK**.
10. Drag your original figure or point **B** and notice how the images are affected.
    What two transformations are used in a glide reflection? ____________ / ____________

Set up your glide reflection as a single transformation.

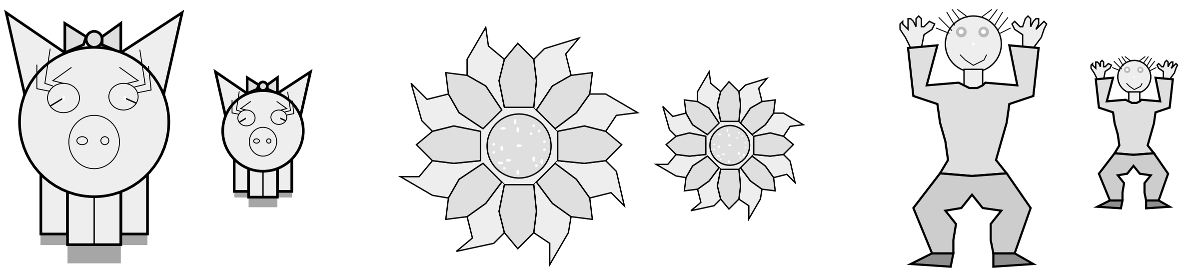
11. Hide the reflected polygon so that just the original polygon (the preimage) and the glide-reflected polygon (the final image) remain.
12. Select your preimage and final image.
13. Go to the **Transform** menu and choose **Define Transform**.
14. Type in Glide Reflection. Then click on **Define**.

You can use the glide reflection you defined for other figures anytime within this sketch. Try the following steps.

15. Hide all images except the preimage.
16. Select the polygon interior of the preimage.
17. Go to the **Transform** menu and choose **Glide Reflection**. While this new image is selected go to the **Transform** menu and choose **Glide Reflection** again. Repeat to construct more glide-reflected images.

On Your Own: Open a new sketch and create your own custom transformation. Make two copies of your sketch. One copy should show all stages. The other copy should have all objects hidden except your original polygon and the final image.

Dilations



A *dilation* is a transformation that shrinks or enlarges a figure. (Unlike other transformations, the *image* is not necessarily the same size as the *preimage*.) Use Sketchpad to investigate dilations.

1. Open the sketch Dilations 1 (Mac) or Dilatns1.gsp (Windows).
2. Dilate $\triangle$**ABC** by using a scale factor.
   a. Using the Selection Arrow tool , click on point **D**. Go to the **Transform** menu and choose **Mark Center**.
   b. Use a selection marquee to select $\triangle$**ABC**. Go to the **Transform** menu and choose **Dilate**.
   c. When the **Dilate** dialog box appears, for Scale Factor enter 3 for New and 4 for Old. Click on **OK**.

   You should see another, smaller triangle appear to the right of the original triangle.
3. Find the ratio of a pair of corresponding sides in the two triangles.
   a. Click on a side of your image (the new triangle). Then click on the corresponding side of the preimage (the original triangle).
   b. Go to the **Measure** menu and choose **Ratio**.

   What ratio appears on your screen for this pair of corresponding sides? __________
4. Follow the directions in step 3 to find the ratio for each pair of corresponding sides of your triangles.

   What is the relationship between the *length of new* and the *length of old* for every pair of corresponding sides? ____
5. Measure an angle of the preimage and a corresponding angle of the image.

   How do the measures compare? ____
6. Find the measures of each pair of corresponding angles in the triangles.
   a. How do the measures of pairs of corresponding angles compare? ____________

   ____
   b. What is the ratio of the measures of any pair of corresponding angles? ________

7. Undo your dilation. Dilate the original figure by the ratio 5 : 4 (5 as New and 4 as Old).

   Answer the following questions. (Verify each by measuring, if needed.)

   a. What is the ratio *length of new : length of old* for each pair of corresponding sides? __________
   b. What is the ratio *measure of new : measure of old* for each pair of corresponding angles? __________

8. Explain in your own words how the corresponding sides and angles of the preimage and image will relate if you create the image by a fixed ratio of 2 : 3 (2 as New and 3 as Old.)

   ____

   ____

9. Open the sketch Dilations 2 (Mac) or Dilatns2.gsp (Windows).

10. Dilate quadrilateral **ABCD** by a marked ratio.

    a. Using the Selection Arrow tool, click on point **P**. Go to the **Transform** menu and choose **Mark Center**.
    b. Holding down the Shift key, click on $\overline{\mathbf{EF}}$ and then on $\overline{\mathbf{GH}}$. Go to the **Transform** menu and choose **Mark Ratio**.
    c. Use a selection marquee to select quadrilateral **ABCD**. Go to the **Transform** menu and choose **Dilate**.
    d. When the **Dilate** dialog box appears, make sure **By Marked Ratio** is chosen. Then click on **OK**. You should see another, smaller quadrilateral appear to the right of the original quadrilateral.

11. Holding down the Shift key, click on $\overline{\mathbf{EF}}$ and then on $\overline{\mathbf{GH}}$. Go to the **Measure** menu and choose **Ratio**.

12. Click on a side of your image (the new quadrilateral). Then click on the corresponding side of the preimage (the original quadrilateral). Go to the **Measure** menu and choose **Ratio**.

    How does the ratio of the side lengths compare to the marked ratio? ____________

    ____

13. Repeat step 12 to find the ratio for each pair of corresponding sides of your quadrilaterals.

    Is the ratio *length of new : length of old* the same for every pair of corresponding sides? ____________

14. Measure an angle of the preimage and a corresponding angle of the image. How do the measures compare? ______________________________

15. Find the measures of each pair of corresponding angles in the quadrilaterals. Answer the following questions.
    a. How do the measures of pairs of corresponding angles compare?

    ____

    b. What is the ratio of the measures of any pair of corresponding angles?

    ____

    c. Are these results consistent with the results you got using the first method for dilating? ____________ Explain why or why not. ______________________

    ____

16. Drag one or more endpoints of $\overline{EF}$ and $\overline{GH}$. Answer the following questions.
    a. What is the effect on the figures and the ratio when $\overline{EF}$ is shorter than $\overline{GH}$?

    ____

    b. What is the effect on the figures and the ratio when $\overline{EF}$ is longer than $\overline{GH}$?

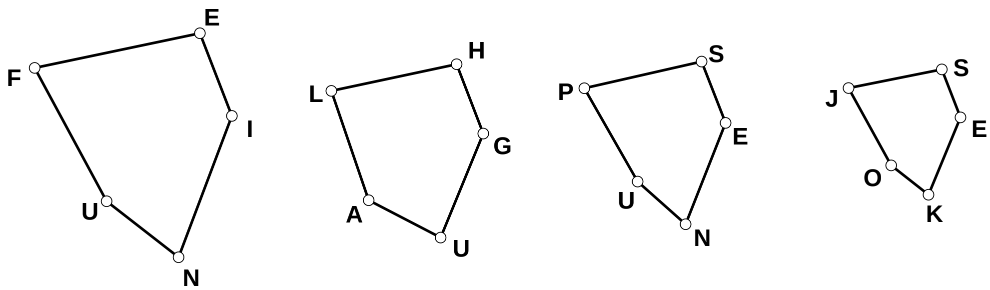
    ____

    c. What is the effect on the figures and the ratio when $\overline{EF}$ is equal in length to $\overline{GH}$?

    ____

On Your Own: Open a new sketch. Draw an angle and a circle. Dilate each, using both methods you have learned. Create a label for each pair showing the ratio of corresponding measures. For the angles, compare their measures in degrees. For the circles, compare the measures of their radii. Put your name on the sketch and print out a copy to turn in.

Similar Figures



Use Sketchpad to explore similar polygons.

1. Open the sketch Similarity 1 (Mac) or Similar1.gsp (Windows).
2. Quadrilateral **TINY** is similar to quadrilateral **LARG**. With this information in mind, answer each question below.
   a. List the pairs of corresponding sides in quadrilateral **TINY** for each side of quadrilateral **LARG**.

      $\overline{LA}$ ____________, $\overline{AR}$ ____________, $\overline{RG}$ ____________, $\overline{GL}$ ____________

   b. What is the ratio of the lengths of each pair of corresponding sides in the quadrilaterals? ____________
   c. List the pairs of corresponding angles in **TINY** for each angle in **LARG**.

      ∠**GLA** __________, ∠**LAR** __________, ∠**ARG** __________, ∠**RGL** __________

   d. What is the relationship of the measures of each pair of corresponding angles in the quadrilaterals? ____
3. Open the sketch Similarity 2 (Mac) or Similar2.gsp (Windows).
4. Hexagon **FLYWEB** is similar to two of the other hexagons shown, but not to the third. Use the ratios of corresponding sides and the measures of corresponding angles to determine which hexagon is *not* similar to **FLYWEB.** Which hexagon is *not* similar to the others? ____________ Explain how your measures show that you are correct.

   ____

   ____

   ____
5. Open the sketch Similarity 3 (Mac) or Similar3.gsp (Windows).

6. Pentagon **FUNIE** is similar to two of the other pentagons shown, but not to the third. Use ratios of corresponding sides and measures of corresponding angles to find the pentagon that is *not* similar. Which pentagon is *not* similar to the others? ________
Explain how your measures show that you are correct. ________________________

____

____

7. Open the sketch Similarity 4 (Mac) or Similar4.gsp (Windows).

8. Since corresponding sides in similar figures have the same ratio and corresponding angles have the same measure, you can often find the lengths of unknown sides and measures of unknown angles. Triangle **HAN** is similar to △**FUT**. With this information in mind, answer each question below.
   a. Measure the ratio of $\overline{\mathbf{HN}}$: $\overline{\mathbf{FT}}$. What is this ratio? ___________
   b. What is the ratio of the lengths of any two corresponding sides? ___________
   c. If $\overline{\mathbf{HN}}$ is 6 inches long, how long is $\overline{\mathbf{FT}}$? (You may use a proportion to solve.) ___________ How do you know you are correct? ________________________

   ____

   ____

   d. Measure ∠**AHN**. What is this measure? ___________
   e. Without measuring, what is the measure of ∠**UFT**? ___________ How do you know you are correct? ____

   ____

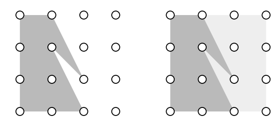
   ____

Mission: Matched Pairs

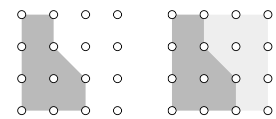
1. Open the sketch Matched Pairs (Mac) or MatchPr.gsp (Windows).

   Your mission, should you choose to accept it, is to work together to divide the four-by-four square array shown into two congruent regions that would fill the square. Do this by connecting points shown in the array to separate the two regions. Mark one solution on each array. You do not have a new solution if the congruent regions are the same as those in another drawing turned in a different direction.

Two Sample Solutions



Example 1



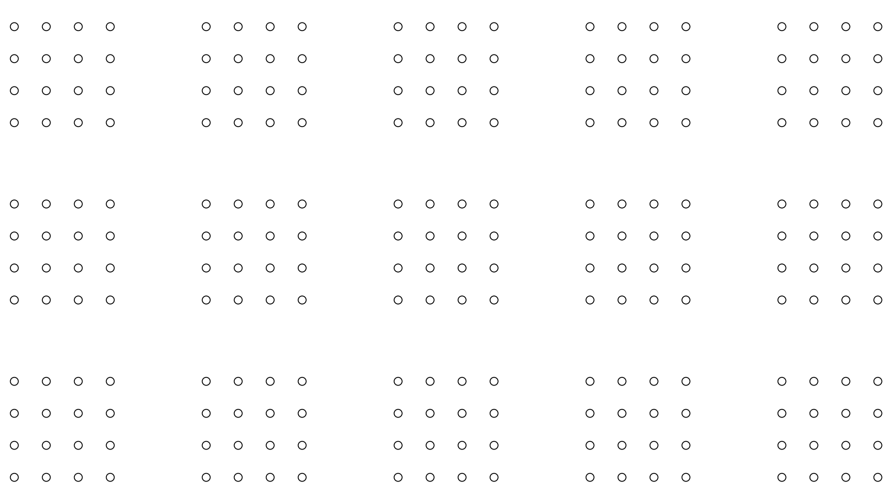
Example 2

2. Check each solution, using the following steps:
   a. Construct the polygon interior on one side of your path.
   b. Select the polygon interior.
   c. Use the appropriate transformation(s) to move the selected polygon so that it fits exactly onto the other half of the array. You may use two reflections or a rotation of 180°. If you choose to rotate, you may double click on the Show Centers button to display the appropriate point to mark as a center for each square.

   See how many different solutions you can find. First work alone. Then compare, check, and combine your findings with those of your partner.

   Use the grids to show up to six solutions. If you need more grids, open the sketch Matched Pairs (Mac) or MatchPr.gsp (Windows) again. If you still have the Matched Pairs sketch open, the new one will open as Untitled. Put your name on each sketch and print out a copy to hand in.

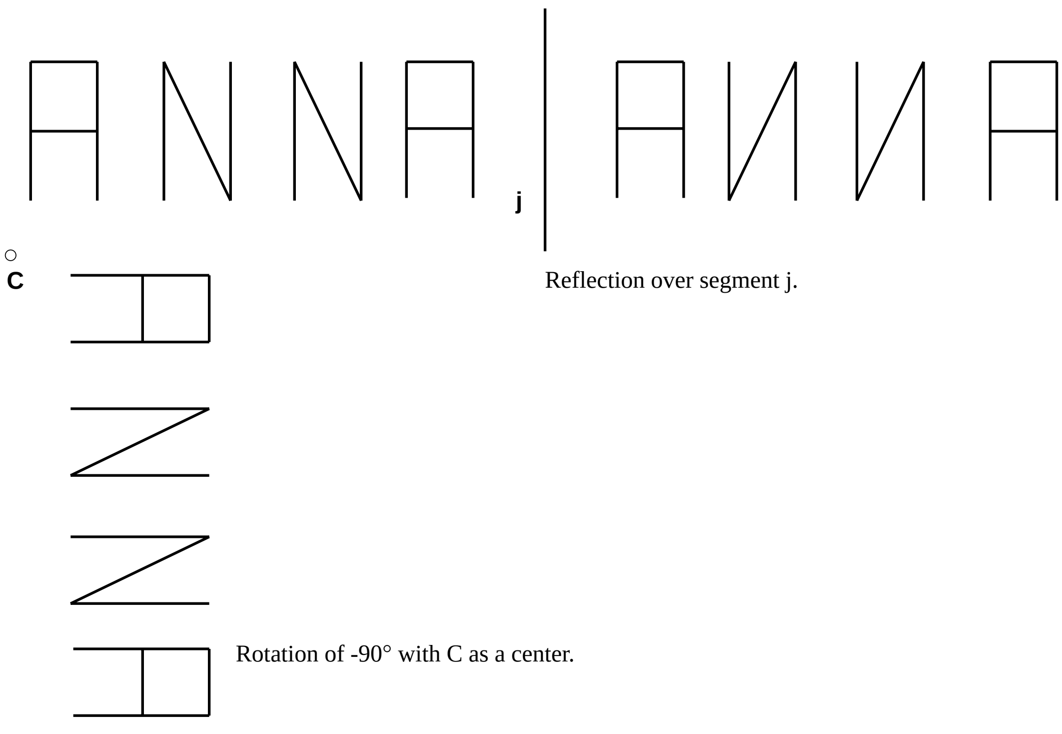
   You may use the grids below to sketch your solutions.



The Name Game

1. Open a new sketch and use the Segment tool to write your first name or nickname. In the same sketch, create each of the following:
   a. a reflection of your name (show your line of reflection, too)
   b. a rotation of your name (show your marked center, too)
      Note: A rotation may be done by a marked angle (show the angle if you use this method).

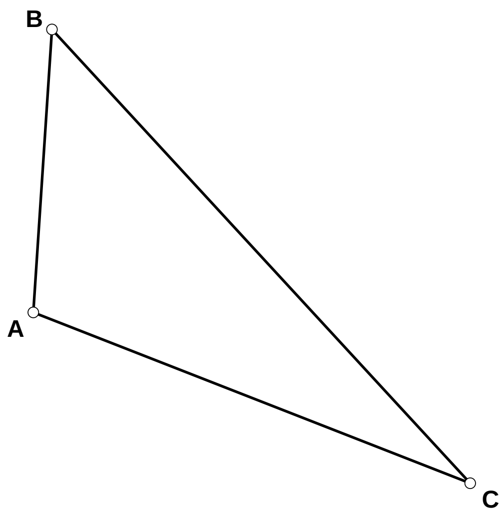
   Be sure the images do not overlap!
2. For each transformation, include a caption similar to the ones shown in the example below.

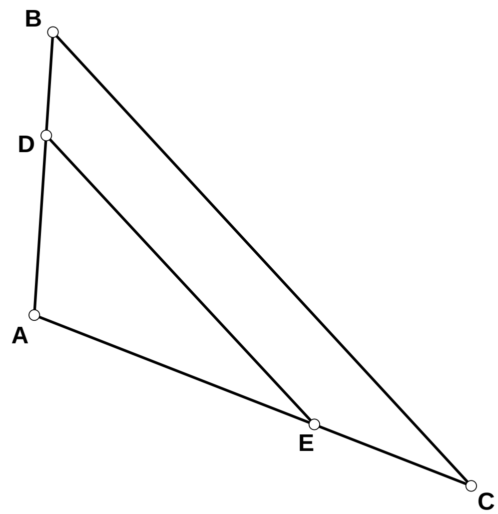


Bonus

You may earn bonus points if you show a translation of your name by using **By Rectangular Vector**. Again, be sure the images do not overlap!

Triangle Buddies





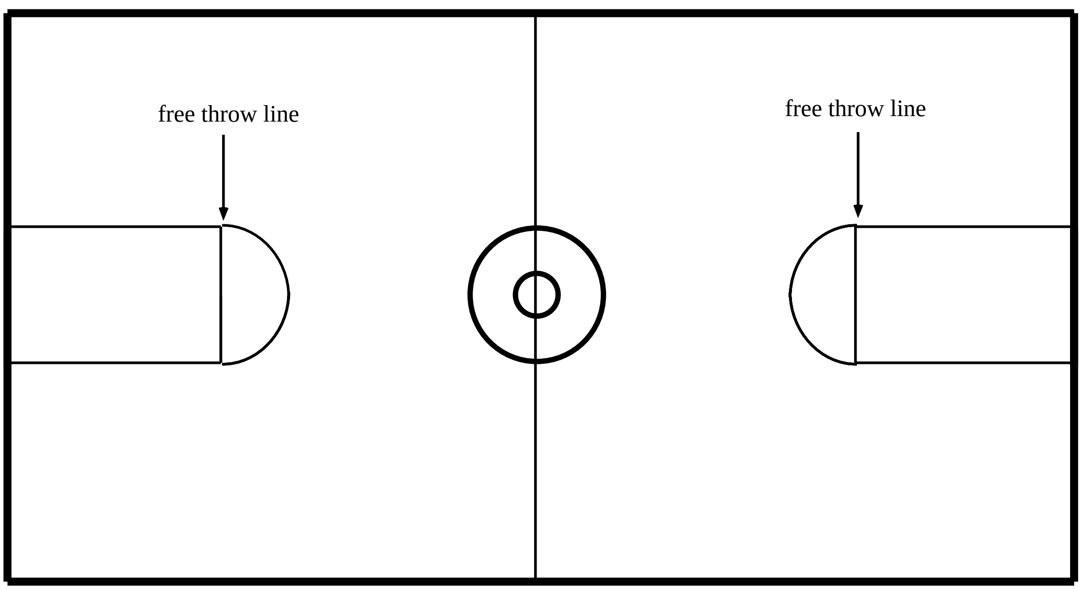
1. Open a new sketch and construct a triangle.
2. Construct a line passing through two sides of the triangle parallel to the third side.
3. Use Sketchpad to see whether the two triangles you now have (Δ**ABC** and Δ**ADE** in the example above) are similar. Drag your triangle to see whether your conclusion holds true for any triangle.
4. Make captions showing appropriate measures and ratios and a statement of your conclusions. Put your name on your sketch, print it out, and hand it in.
5. Repeat steps 2 and 3 with a line parallel to a different side of your triangle.

Go, Team, Go

In a *scale drawing,* the ratio of the measures of real objects to those in the drawing is always the same. For example, if the scale for a drawing is 3 cm to 2 m, an object that has a length of 3 cm in the scale drawing has an actual length of 2 m, and the ratio of any two measures of *object in drawing : real object* is 3 : 200.

1. Your group is to measure the basketball court at your school and use Sketchpad to make a scale drawing. Be sure to show the free throw line and lane, the center circles, and all other lines on the court.
2. Put a title, your names, and the scale for your drawing on your sketch. Print out a copy and hand it in.

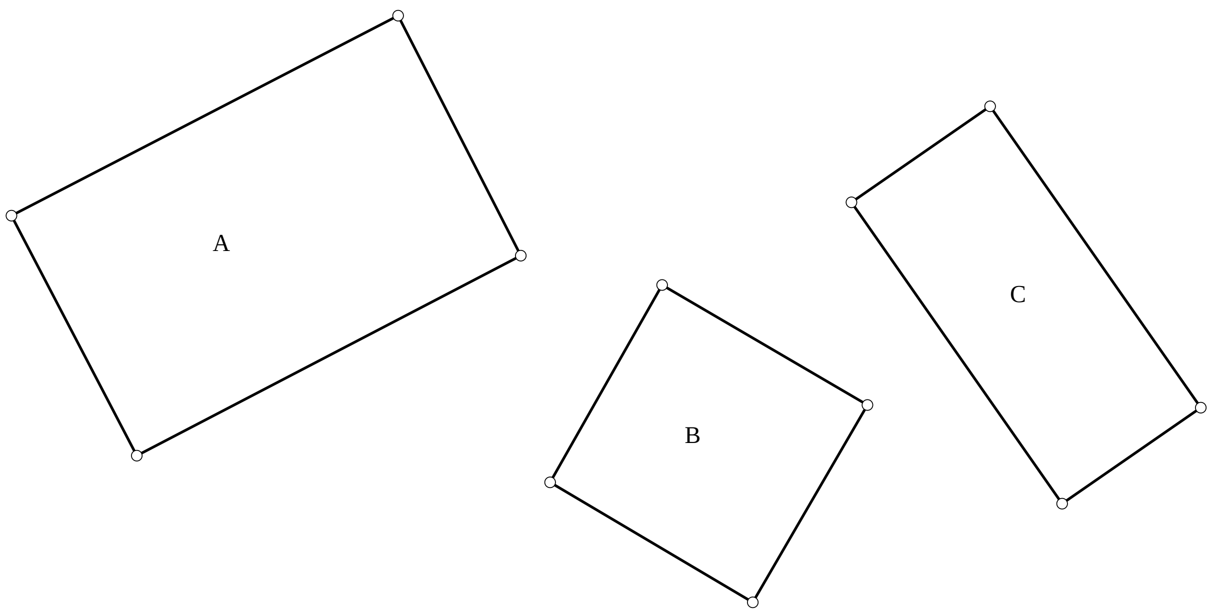
Basketball Court



Golden Rectangle Survey

A *golden rectangle* is one in which the ratio of the length (the longer side) to the width creates a shape that supposedly is particularly pleasing. This ratio is referred to as the *golden ratio* and is present when

$$\frac{\text{length}}{\text{width}} = \frac{(\text{length} + \text{width})}{\text{length}}$$



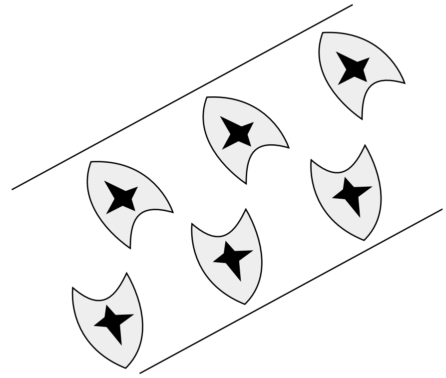
1. Use Sketchpad to construct a golden rectangle.
2. In the same sketch, construct three other rectangles that are *not* golden rectangles. (Do not make the ratio *length : width* close to the golden ratio in your other rectangles.) Print out your sketch.
3. Ask 20 people to tell you which rectangle is the most pleasing. Record their choices.
4. Combine your results with those of your classmates. Did most people choose the golden rectangle as the most pleasing shape?

Think It Over

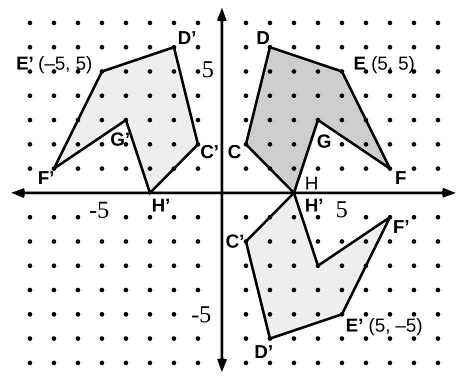
5. Are all the golden rectangles created by you and your classmates similar? Why or why not? Verify your answer using Sketchpad.
6. If you dilate your golden rectangle, will the new figure also be a golden rectangle? Why or why not? Verify your answer using Sketchpad.

More Ideas for Transformations Projects

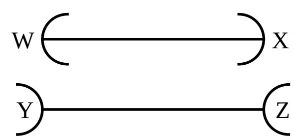
1. Look at designs in wallpaper books or on fabric to find an example of a strip symmetry made using glide reflections. Create a Sketchpad version of the design. Make a poster showing the wallpaper or fabric sample, your sketch, and an explanation of how glide reflections were used to create the design.



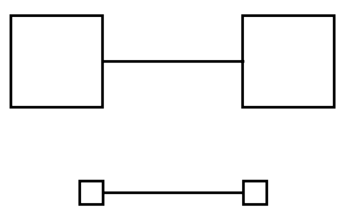
2. Explore the general effects of reflecting figures on the coordinate plane over the x- and y-axes. Take particular notice of the coordinates of points in the preimage and image. Can you find any patterns that would allow you to predict the coordinates in the image before you reflect? Make any conjectures you can. Drag the points in your figures to test your ideas.



3. Use your knowledge of transformations and similar figures to create several optical illusions. You may want to do some research in your school library or on the Internet for ideas.

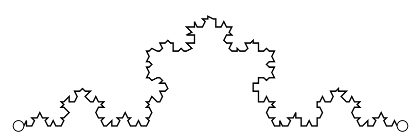


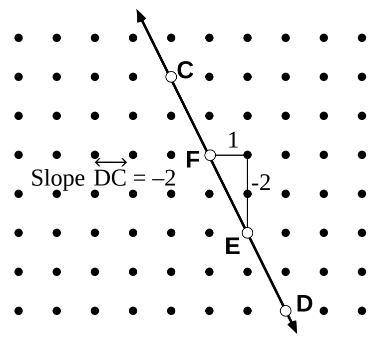
Which is greater in length: $\overline{WX}$ or $\overline{YZ}$?



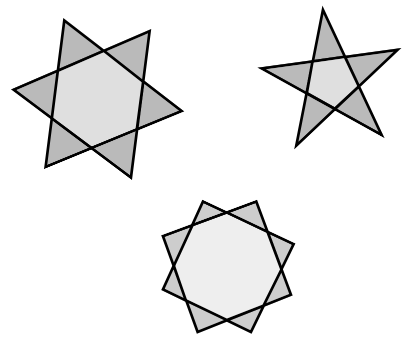
Are the lines connecting the pairs of squares equal in length?

4. Use transformations to create an original fractal. You may want to begin by completing one of the fractals in the Constructions unit of this book and/or investigating the topic on your own. Be sure you use a Sketchpad script to create the stages of your fractal.



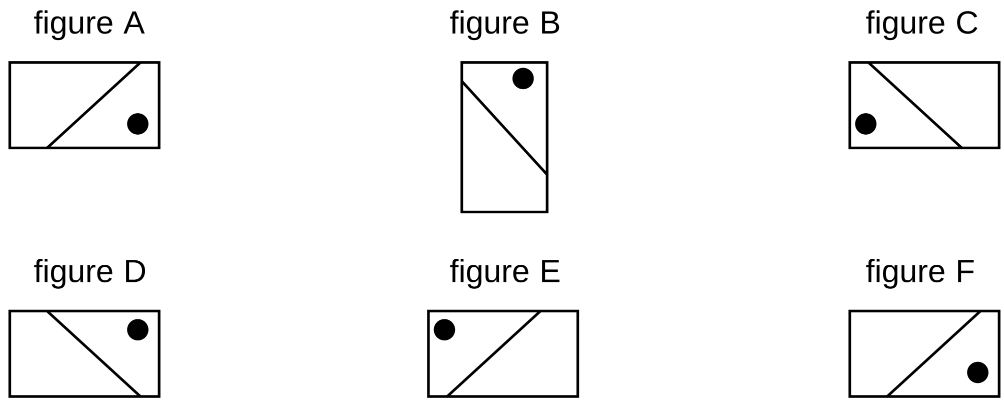


5. Investigate the concept of slope of lines in the graphs of functions on the coordinate plane. Prepare a presentation using sketches you create with Sketchpad to explain slope using translations. For example, translating a point 1 unit horizontally and 2 units vertically several times will produce points on a line with slope –2.



6. Create star polygons with Sketchpad using transformations. Try to create each one using at least two different methods.

1. Determine which figures are images of figure A under the transformations named in a–c below.

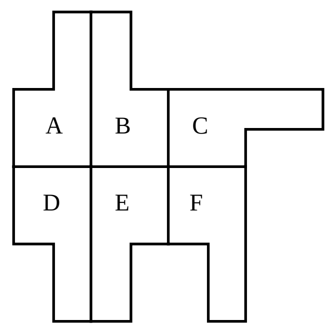


   a. Which figure(s) are images of figure A after *one* reflection? ________________

   b. Which figure(s) are images of figure A after a rotation? ___________________

   c. Which figure(s) are images of figure A after a translation? __________________

2. Give one transformation (reflection, rotation, or translation) that will move the preimage (the first figure listed) onto the image (the second figure listed).

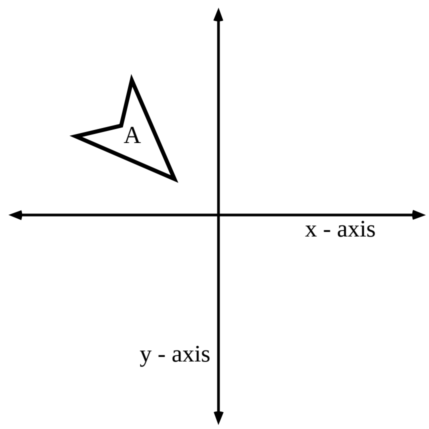


   a. Move figure A onto figure B. _____________
   b. Move figure B onto figure C. _____________
   c. Move figure B onto figure D. _____________
   d. Move figure A onto figure E. _____________
   e. Move figure D onto figure F. _____________

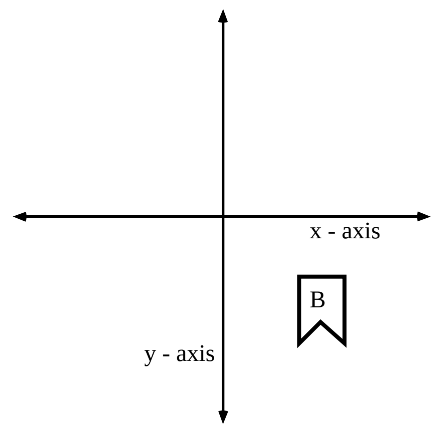
3. Draw each image that results from the given transformation.

   a.

   b.

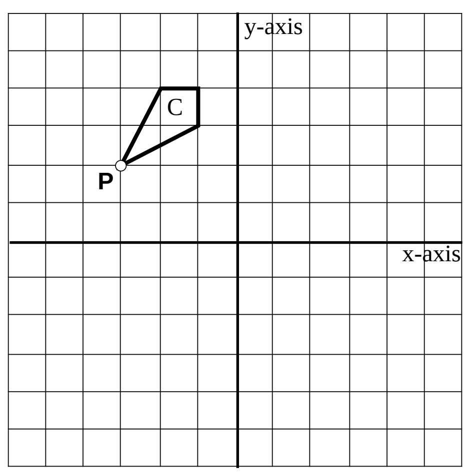


Reflect figure A over the y-axis.



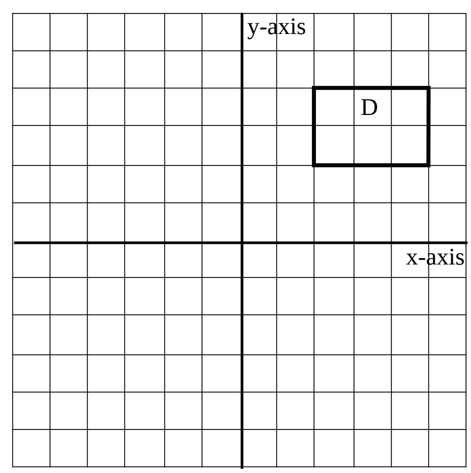
Reflect figure B over the x-axis.

c.



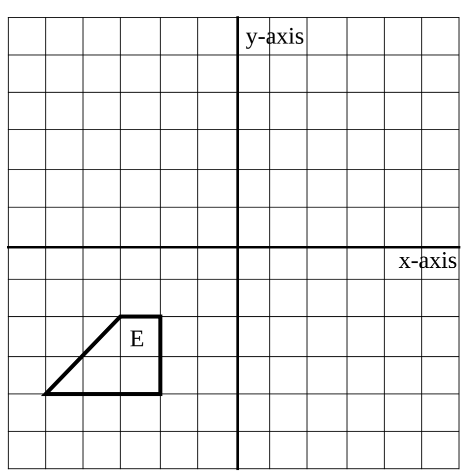
Rotate figure C 90° clockwise around point P.

d.

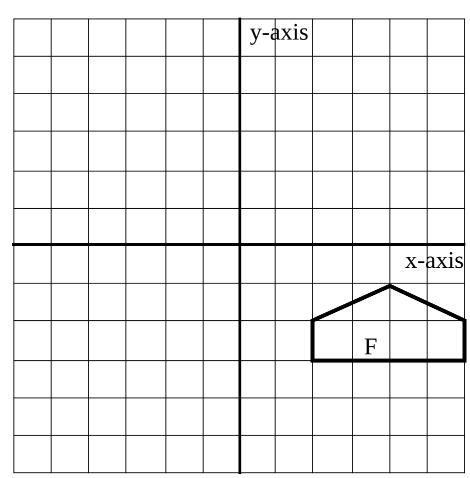


Rotate figure D 180° around the origin.

e.



f.

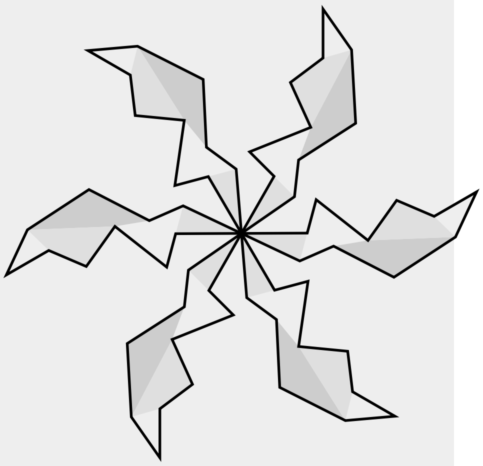


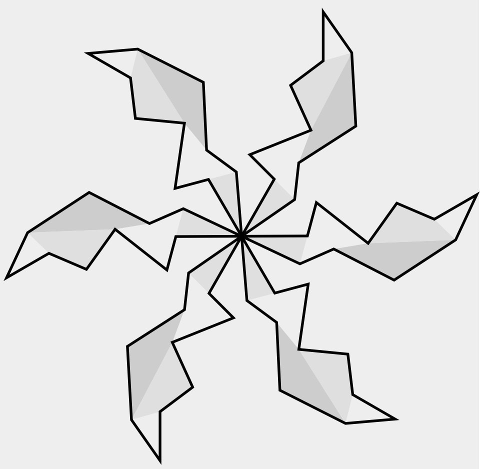
Translate figure E by $\langle 6, 4 \rangle$.

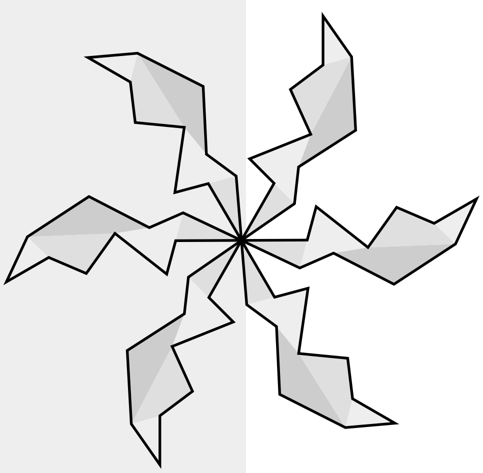
Translate figure F by $\langle -7, 5 \rangle$.

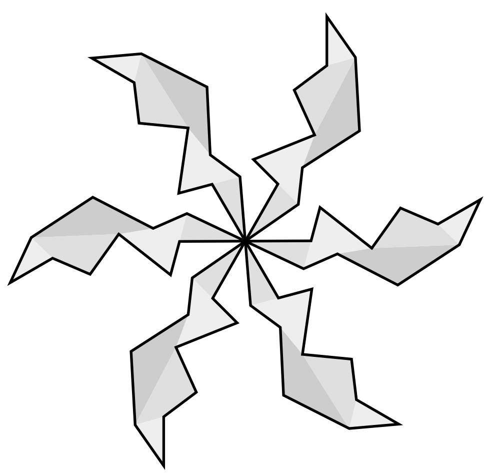
Unit 7

CONSTRUCTIONS









Commentary

This unit provides students with step-by-step instructions that can be used with The Geometer's Sketchpad to produce figures that maintain their essential characteristics. Unlike a drawing, each of the finished products can be dragged and will not "fall apart" or lose its defining properties. In addition, the products are dynamic and allow further exploration, an advantage Euclid and others did not have!

The purpose of these activities is to introduce middle-school students to the capabilities of Sketchpad as an alternative to compass and straightedge constructions and to provide experiences that will spark their imaginations to do constructions of their own. We believe they also provide an intuitive base for the constructions and formal proofs the students will encounter later in their mathematics instruction.

Each construction is designed to stand alone; that is, none is dependent upon having done a previous activity, although, of course, related experiences will certainly enhance the benefits. Some of the constructions fit nicely into other units of the book, and for each of these a reference is included in the Teacher Notes of the appropriate unit. For example, using **Constructing an Equilateral Triangle** will be appropriate for some students as they work through the Triangles unit. Other constructions may fit appropriately in two or more places; for example, **Constructing a Golden Rectangle** might be used with some groups during a study of quadrilaterals, while for others it might fit best after a study of transformations and similar polygons.

Our intent is that these constructions serve as examples and motivators to middle-school students. We love to see their follow-up constructions. Our experience with Sketchpad gives us confidence that middle-school students can grasp the concepts of sound constructions and create their own. Give them the tools and a little experience, then step back and get ready for exciting results!

For each construction, the major steps required have been delineated. Detailed instructions are provided for each step and are designated by the compass symbol. Experienced Sketchpad users may not need the detailed instructions included with each step. This format allows such students to proceed at their own pace while providing a foolproof method for less experienced students to have success as well. Don't forget to emphasize the wonderful **Undo** feature of Sketchpad. These activities reinforce that favorite adage of teachers: "Perseverance pays off!"

As they use the software, your students may be eager to use shortcuts with Sketchpad. Some users are skilled enough to use shortcuts almost immediately, while others are not sufficiently proficient with either Sketchpad or using a mouse until much later. We certainly believe that any child should be allowed to use any shortcut whenever he or she demonstrates a level of proficiency that minimizes errors. However, it has been our experience with middle-school students that it works best to postpone the use of shortcuts with the tools until the students fully understand the steps each shortcut replaces, as discussed in the introduction to this book. For your convenience, we have included a guide to shortcuts with Sketchpad at the end of this unit.

We suggest that you remind students who have minimal Sketchpad experience that to select an object or a figure one usually clicks on the object with the Selection Arrow tool or drags a selection marquee around it.

We believe that including some exposure to constructions in a middle-school geometry curriculum is important. Going through such a process adds a dimension to students' mathematical experience that is enriching and, to many, quite motivating as well.

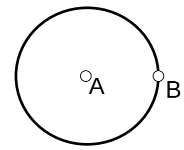
Constructing an Equilateral Triangle

An *equilateral triangle* is a triangle with all three sides the same length. Labels are shown to make the directions clearer, but you don't need labels in your drawing. Whenever needed, use the detailed instructions marked by . *Make sure you did each step correctly before you go on to the next step.*

Step 1: Open a new sketch and construct a circle with radius **AB**.

a. Go to the **File** menu and choose **New Sketch**.

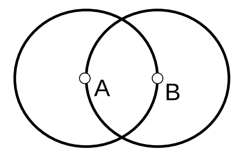
b. Click on the Circle tool to highlight it. Drag to draw a circle in your sketch.



Step 2: Construct a circle with center **B** and radius **BA**.

a. Click on the Selection Arrow tool . Click in any blank space to deselect objects.

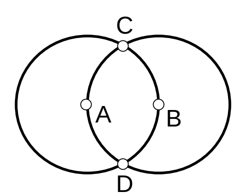
b. Hold down the Shift key. Select point **B** and then point **A**. Go to the **Construct** menu and select **Circle By Center+Point**.



Step 3: Construct intersection point **C**.

a. Click in any blank space to deselect objects.

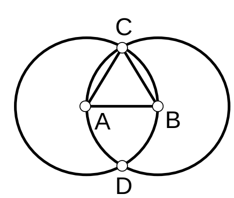
b. Hold down the Shift key. Select circle **A** and then circle **B** (select the circles, not points **A** and **B**). Go to the **Construct** menu and choose **Point At Intersection**. Two intersection points will appear. We will use point **C**.



Step 4: Construct segments **AB**, **AC**, and **CB**.

a. Click in any blank space to deselect objects.

b. Hold down the Shift key. Select points **A**, **B**, and **C**. Go to the **Construct** menu and choose **Segment**.

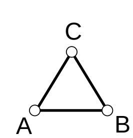


Step 5: Hide the two circles and point **D**.

a. Click in any blank space to deselect objects.

b. Hold down the Shift key. Select the two circles and point **D**.

c. Go to the **Display** menu and choose **Hide Objects**.



Congratulations! You have a new Sketchpad equilateral triangle. Unlike an equilateral triangle that you might draw using only the Segment tool, this equilateral triangle stays equilateral when you drag it. See what happens when you drag any of its vertices. Why do you think dragging point **A** or **B** gives a different result than dragging point **C**?

Teach Sketchpad This Construction

1. Use a selection marquee to select your equilateral triangle. (Make sure all three sides and vertices are selected.)
2. Go to the **Work** menu and choose **Make Script**.
3. Go to the **File** menu and choose **New Sketch**. Using the Point tool, construct two points in your sketch.
4. Select both points, click back on your **Script** window, and click on the Fast button. You should see an equilateral triangle constructed in your sketch!
5. Click back on the **Script** dialog box, go to the **File** menu, and choose **Save**. Call this script Equilateral Script (Mac) or Equitri.gss (Windows). Save the script to a disk or folder according to your teacher's directions. Any time in the future you need an equilateral triangle, simply play your script!

Follow-up Construction

Construct an isosceles triangle.

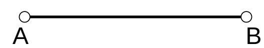
Constructing a Parallelogram

A *parallelogram* is a quadrilateral with both pairs of opposite sides parallel. Follow the directions below to construct a parallelogram. Labels are shown to make the directions clearer, but you don't need labels in your drawing. Whenever needed, use the detailed instructions marked by the compass icon. *Make sure you did each step correctly before you go on to the next step.*

Step 1: Open a new sketch and construct $\overline{AB}$.

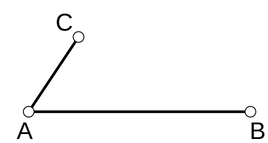
a. Go to the **File** menu and choose **New Sketch**.

b. Use the Segment tool and drag to draw a horizontal segment in your sketch.



Step 2: Construct $\overline{AC}$.

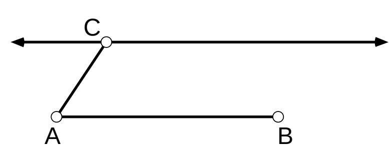
a. Click on point **A**, and drag to construct $\overline{AC}$.



Step 3: Construct a line through point **C**, parallel to $\overline{AB}$.

a. Click on the Selection Arrow tool. Click in any blank space to deselect objects.

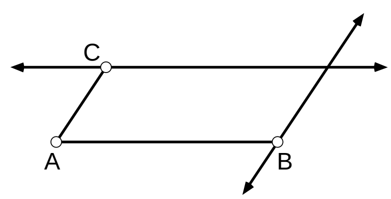
b. Hold down the Shift key. Select point **C** and $\overline{AB}$. Go to the **Construct** menu and choose **Parallel Line**.



Step 4: Construct a line through point **B** parallel to $\overline{AC}$.

a. Click in any blank space to deselect objects.

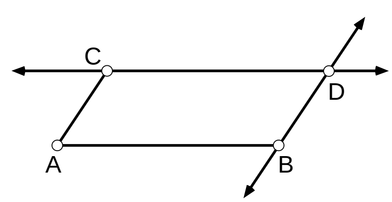
b. Hold down the Shift key. Select point **B** and $\overline{AC}$. Go to the **Construct** menu and choose **Parallel Line**.



Step 5: Construct intersection point **D**.

a. Click in any blank space to deselect objects.

b. Hold down the Shift key. Select each of the two lines. Go to the **Construct** menu and choose **Point At Intersection**.



Step 6: Hide the lines.

a. Click in any blank space to deselect objects.

b. Hold down the Shift key. Select $\overleftrightarrow{BD}$ and $\overleftrightarrow{CD}$. Go to the **Display** menu and choose **Hide Lines**.

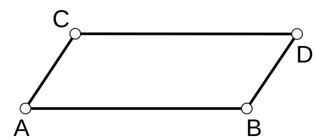
Step 7: Construct sides $\overline{CD}$ and $\overline{DB}$.

a. Click in any blank space to deselect objects.

b. Hold down the Shift key. Select points **C** and **D**. Go to the **Construct** menu and choose **Segment**.

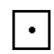
c. Click in any blank space to deselect objects.

d. Hold down the Shift key. Select points **D** and **B**. Go to the **Construct** menu and choose **Segment**.



Congratulations! You have a new Sketchpad parallelogram. Unlike a parallelogram that you might draw using only the Segment tool , this parallelogram stays a parallelogram when you drag it. See what happens when you drag any of its vertices. Why do you think dragging point **A**, **B**, or **C** gives a different result than dragging point **D**? See whether you can drag a part of your parallelogram so that it appears to be a rectangle, a square, or a rhombus.

Teach Sketchpad This Construction

1. Use a selection marquee to select your parallelogram.
2. Go to the **Work** menu and choose **Make Script**.
3. Go to the **File** menu and choose **New Sketch**. Using the Point tool , construct three points in your sketch.
4. Select all three points, click back onto your **Script** window, and click on the Fast button. You should see a parallelogram constructed in your sketch!
5. Click back on the **Script** window, go to the **File** menu, and choose **Save**. Call this script Parallelogram Script (Mac) or Paragram.gss (Windows). Any time in the future you need a parallelogram, simply play your script!

Follow-up Constructions

Construct any two of the figures listed below. Notice the characteristics given in each definition. Be sure your figure keeps those characteristics when dragged and does not fall apart. Make a script for each of your constructions.

A. *Rhombus*—a quadrilateral with both pairs of opposite sides parallel and all side lengths equal.

B. *Rectangle*—a parallelogram with four right angles.

C. *Square*—a rectangle with all sides congruent, or a rhombus with four right angles.

D. *Trapezoid*—a quadrilateral with exactly one pair of parallel sides.

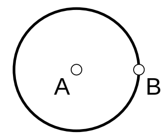
Constructing a Rhombus

A *rhombus* is defined as either a quadrilateral with both pairs of opposite sides parallel and all side lengths equal or as a parallelogram with all sides congruent. Follow the directions below to construct a rhombus. Labels are shown to make the directions clearer, but you don't need labels in your drawing. Whenever needed, use the detailed instructions marked by . *Make sure you did each step correctly before you go on to the next step.*

Step 1: Open a new sketch and construct a circle.

a. Go to the **File** menu and choose **New Sketch**.

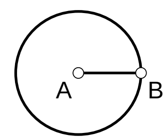
b. Click on the Circle tool . Drag in your sketch to draw a circle.



Step 2: Construct radius $\overline{\textbf{AB}}$.

a. Click on the Selection Arrow tool . Click in any blank space to deselect objects.

b. Hold down the Shift key. Select points **A** and **B**, go to the **Construct** menu, and choose **Segment**.

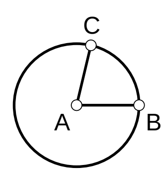


Step 3: Construct $\overline{\textbf{AC}}$, where **C** is any point on the circle.

a. Click in any blank space to deselect objects.

b. Select the circle (not a point on the circle). Go to the **Construct** menu and choose **Point On Object**.

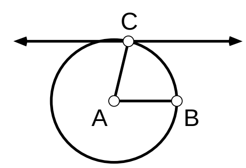
c. Hold down the Shift key. Select points **A** and **C**. Go to the **Construct** menu and choose **Segment**.



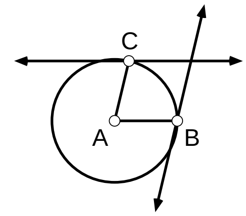
Step 4: Construct a line through point **C** parallel to $\overline{\textbf{AB}}$.

a. Click in any blank space to deselect objects.

b. Hold down the Shift key. Select point **C** and $\overline{\textbf{AB}}$. Go to the **Construct** menu and choose **Parallel Line**.



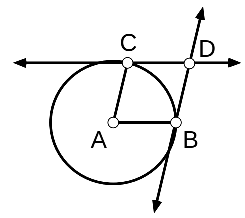
Step 5: Construct a line through point **B**, parallel to $\overline{\mathbf{AC}}$.



a. Click in any blank space to deselect objects.

b. Hold down the Shift key. Select point **B** and $\overline{\mathbf{AC}}$. Go to the **Construct** menu and choose **Parallel Line**.

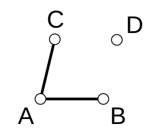
Step 6: Construct intersection point **D**.



a. Click in any blank space to deselect objects.

b. Hold the Shift key down. Select each of the two lines. Go to the **Construct** menu and choose **Point At Intersection**.

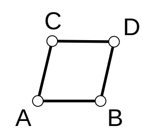
Step 7: Hide the circle and $\overleftrightarrow{\mathbf{CD}}$ and $\overleftrightarrow{\mathbf{BD}}$.



a. Click in any blank space to deselect objects.

b. Hold down the Shift key. Select the circle (not a point on the circle) and $\overleftrightarrow{\mathbf{CD}}$ and $\overleftrightarrow{\mathbf{BD}}$. Go to the **Construct** menu and choose **Hide Objects**.

Step 8: Construct $\overline{\mathbf{CD}}$ and $\overline{\mathbf{BD}}$.



a. Click in any blank space to deselect objects.

b. Hold down the Shift key. Select points **C** and **D**. Go to the **Construct** menu and choose **Segment.**

c. Click in any blank space to deselect objects. Hold down the Shift key. Select points **B** and **D**. Go to the **Construct** menu and choose **Segment.**

Congratulations! You have a new Sketchpad rhombus. Unlike a rhombus that you might draw using only the Segment tool, this rhombus stays a rhombus when you drag it. See what happens when you drag any of its vertices. Why do you think dragging point **A**, **B**, or **C** gives a different result than dragging point **D**? See whether you can drag a part of your rhombus so that it appears to be a square.

Teach Sketchpad This Construction

1. Use a selection marquee to select your rhombus.
2. Go to the **Work** menu and choose **Make Script**.

3. Go to the **File** menu and choose **New Sketch**. Using the Point tool [•], construct two points in your sketch.
4. Select both points, click back onto your **Script** window, and click on the Fast button. You should see a rhombus constructed in your sketch!
5. Click back on the **Script** window, go to the **File** menu, and choose **Save**. Call this script Rhombus Script (Mac) or Rhombus.gss (Windows). Any time in the future you need a rhombus, simply play your script!

Follow-up Constructions

Construct any two of the figures listed below. Notice the characteristics given in each definition. Be sure your figure keeps those characteristics when dragged and does not fall apart. Make a script for each of your constructions.

A. *Parallelogram*—a quadrilateral with two pairs of parallel sides, or a quadrilateral whose opposite sides are congruent.

B. *Rectangle*—a parallelogram with four right angles.

C. *Square*—a rectangle with all sides congruent, or a rhombus with four right angles.

D. *Trapezoid*—a quadrilateral with exactly one pair of parallel sides.

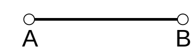
Constructing a Golden Rectangle

A *golden rectangle* is a rectangle in which the ratio of the length (the longer side) to the width is the *golden ratio.* This ratio exists when

$$\frac{\text{length}}{\text{width}} = \frac{\text{length + width}}{\text{length}}$$

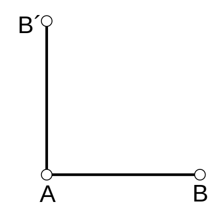
Follow the directions below to construct a golden rectangle. Labels are shown to make the directions clearer, but you don't need labels in your drawing. Whenever needed, use the detailed instructions marked by . *Make sure you did each step correctly before you go on to the next step.*

Step 1: Open a new sketch and construct a square.



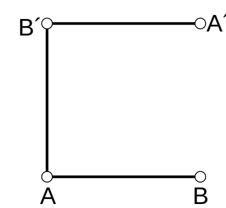
a. Go to the **File** menu and choose **New Sketch**.

b. Click on the Segment tool and construct $\overline{\mathbf{AB}}$.



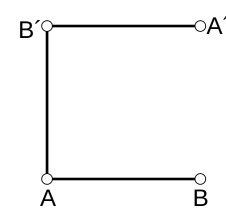
c. Click on the Selection Arrow tool . Select point **A**. Go to the **Transform** menu and choose **Mark Center**.

d. Hold down the Shift key. Select point **B** and $\overline{\mathbf{AB}}$. Go to the **Transform** menu and choose **Rotate**.



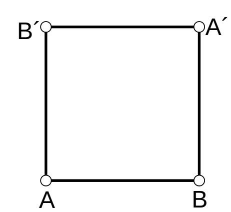
e. Choose **By Fixed Angle** and enter 90. Click on **OK**. Click in any blank space to deselect objects.

f. Select point **B′**. Go to the **Transform** menu and choose **Mark Center**.



g. Hold down the Shift key. Select point **A** and $\overline{\mathbf{AB'}}$. Go to the **Transform** menu and choose **Rotate**.

h. Choose **By Fixed Angle** and enter 90. Click on **OK**. Click in any blank space to deselect objects.

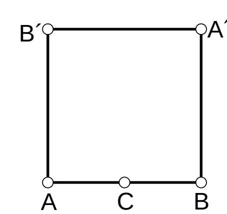


i. Hold down the Shift key. Select points **A′** and **B**. Go to the **Construct** menu and choose **Segment**.

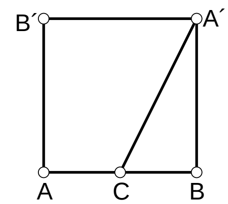
Step 2: Construct midpoint **C** on side $\overline{\mathbf{AB}}$.

a. Click in any blank space to deselect objects.

b. Select $\overline{\mathbf{AB}}$. Go to the **Construct** menu and choose **Point At Midpoint**.



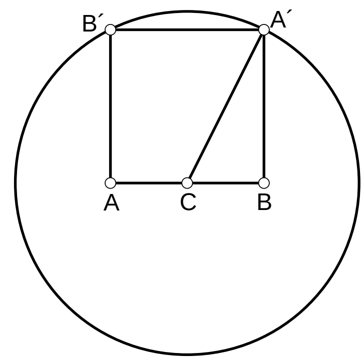
Step 3: Construct $\overline{\mathbf{CA'}}$.



a. Click in any blank space to deselect objects.

b. Hold down the Shift key. Select point **C** and then point **A′**. Go to the **Construct** menu and choose **Segment**.

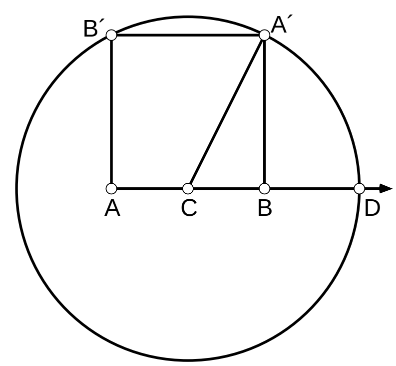
Step 4: Construct a circle with center **C** and radius $\overline{\mathbf{CA'}}$.



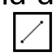
a. Click in any blank space to deselect objects.

b. Hold down the Shift key. Select point **C** and segment $\overline{\mathbf{CA'}}$. Go to the **Construct** menu and choose **Circle By Center+Radius**.

Step 5: Extend side $\overline{\mathbf{AB}}$ to intersect the circle at point **D**.



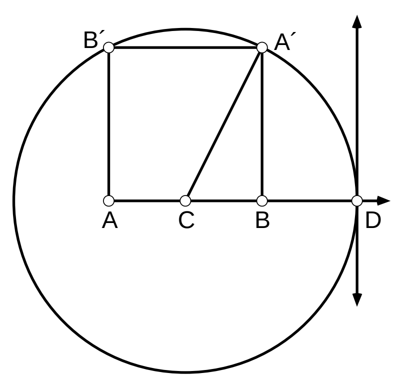
a. Click in any blank space to deselect objects.

b. Press and hold down the mouse button on the Segment tool . Drag to the right and click on the Ray tool  to highlight it.

c. Click on the Selection Arrow tool . Hold down the Shift key. Select point **A** and then point **B**. Go to the **Construct** menu and choose **Ray**.

d. Click in any blank space to deselect objects. Hold down the Shift key. Select $\overrightarrow{\mathbf{AB}}$ and the circle. Go to the **Construct** menu and choose **Point At Intersection**.

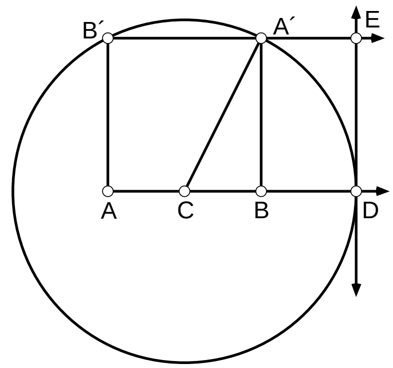
Step 6: Construct a line perpendicular to $\overrightarrow{\mathbf{AB}}$ through **D**.



a. Click in any blank space to deselect objects.

b. Hold down the Shift key. Select $\overrightarrow{\mathbf{AB}}$ and point **D**. Go to the **Construct** menu and choose **Perpendicular Line**.

Step 7: Extend $\overline{B'A'}$ to intersect the perpendicular line at point **E**.

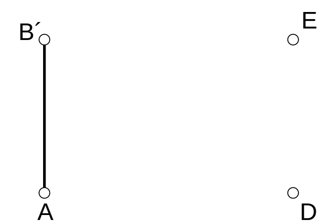


a. Click in any blank space to deselect objects.

b. Hold down the Shift key. Select point **B´** and then point **A´**. Go to the **Construct** menu and choose **Ray**.

c. Hold down the Shift key. Select $\overrightarrow{B'A'}$ and the perpendicular line. Go to the **Construct** menu and choose **Point At Intersection**.

Step 8: Hide all unneeded parts to leave rectangle **AB´ED**.

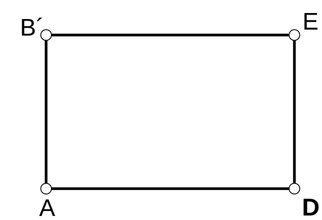


step b

a. Click in any blank space to deselect objects.

b. Hold down the Shift key. Select any unwanted points, segments, rays, and circle. Go to the **Display** menu and choose **Hide Objects**.

c. Press and hold down the mouse button on the Ray tool. Drag to the right and click on the Segment tool to highlight it.



step d

d. Click on the Selection Arrow tool. Hold down the Shift key. Select points **A**, **B´**, **E**, and **D**, *in that order.* Go to the **Construct** menu and choose **Construct Segment**.

Step 9: Measure the sides of rectangle **AB´ED** and find the ratio of length to width.

Calculator (Windows)

a. Click in any blank space to deselect objects.

b. Hold down the Shift key. Select sides $\overline{AD}$ and $\overline{DE}$. Go to the **Measure** menu and choose **Length**.

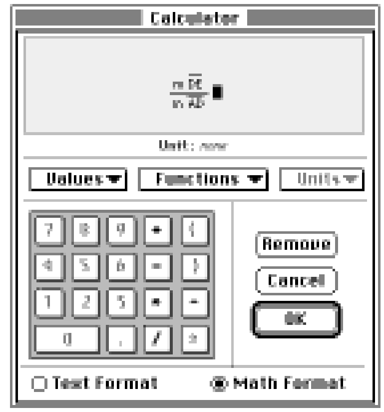
c. Double click on one of the measures to display the calculator.

d. Click on the measure of $\overline{DE}$, click on **/**, click on the measure of $\overline{AD}$, and then click on **OK**.

Step 10: Find the ratio of (length + width) to length of rectangle **AB´ED**.

a. Double click on one of the measures to display the calculator again.

b. Click on **(**, click on the measure of $\overline{DE}$, click on **+**, click on the measure of $\overline{AD}$, click on **)**, click on **/**, click on the measure of $\overline{AD}$, and then click on **OK**.



Calculator (Macintosh)

Drag point **A**. Compare the two ratios. Do the ratios change as you change the size of your rectangle?

____

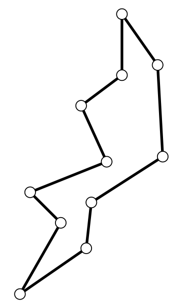
Is each rectangle displayed a golden rectangle?

____

Constructing a Sketchpad Kaleidoscope

Follow the directions below to construct a Sketchpad kaleidoscope. Whenever needed, use the detailed instructions marked by . *Make sure you did each step correctly before you go on to the next step.*

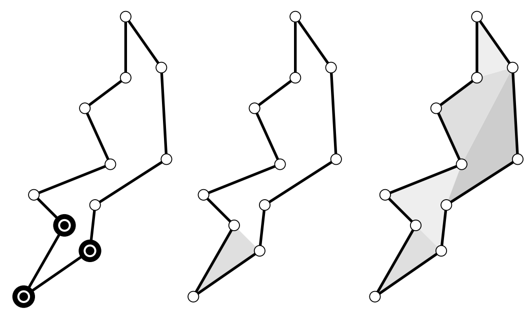
Step 1: Open a new sketch and construct a many-sided polygon.



a. Go to the **File** menu and choose **New Sketch**.

b. Use the Segment tool to construct a polygon with many sides (make it long and somewhat slender).

Step 2: Construct several polygon interiors within your polygon. Shade them different colors.



step b step c step e

a. Click on the Selection Arrow tool . Click in any blank space to deselect objects.

b. Hold down the Shift key. Select three or four points in clockwise or counter-clockwise order.

c. Go to the **Construct** menu and choose **Polygon Interior**.

d. While the polygon interior is still selected, go to the **Display** menu and choose a shade and/or color for your polygon interior.

e. Click in any blank space to deselect objects. Repeat steps b, c, and d until you have constructed several polygon interiors with different colors or shades.

Step 3: Mark the bottom vertex point of your polygon as the center. Hide the points and rotate the polygon by an angle of 60°.

a. Click in any blank space to deselect objects.

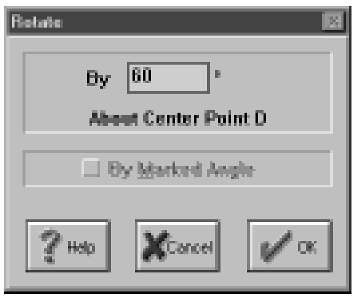
b. Select the bottom vertex point. Go to the **Transform** menu and choose **Mark Center**.

c. Click on the Point tool . Go to the **Edit** menu and choose **Select All Points**. Go to the **Display** menu and choose **Hide Points**.

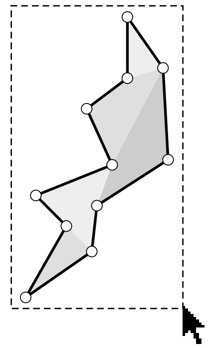
Geometry Activities for Middle School Students with The Geometer's Sketchpad

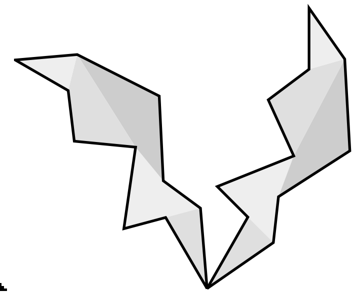
d. Click on the Selection Arrow tool. Use a selection marquee to select your polygon. Go to the **Transform** menu and choose **Rotate**.

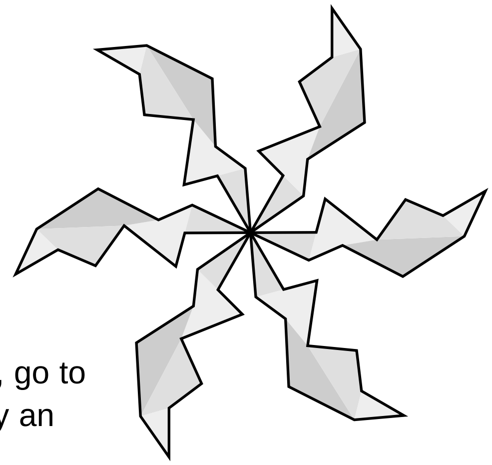
e. Choose **By Fixed Angle**. Enter 60 and then click on **OK**. (Pick a different factor of 360 if you wish.)



Rotate Dialog Box (Windows)


selection marquee


after 60° rotation



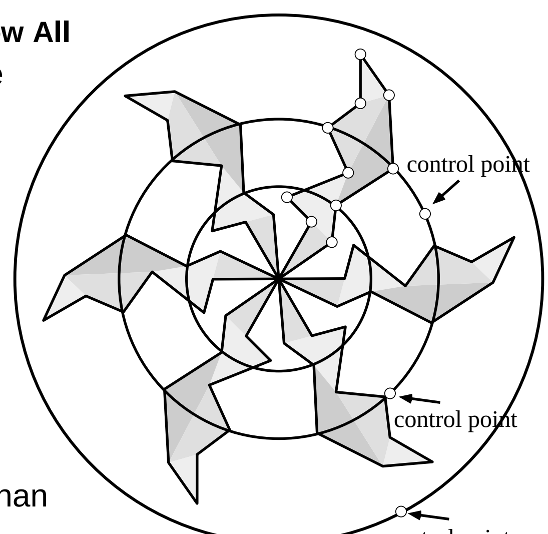
Step 4: Continue to rotate the new rotated images until you have completed your kaleidoscope.

a. While the new rotated image is still selected, go to the **Transform** menu and rotate this image by an angle of 60°. Remember to click on **OK**.

b. When the newer rotated image appears, and while it is still selected, go to the **Transform** menu and rotate this image by an angle of 60°. Remember to click on **OK**.

c. Repeat this process until you have constructed your complete kaleidoscope.

d. Go to the **Display** menu and choose **Show All Hidden.** You should see the points on the original arm reappear.



Step 5: Construct circles with their centers at the center of your kaleidoscope.

a. Click in any blank space to deselect objects.

b. Click on the Circle tool. Press on the center point of your kaleidoscope and drag a circle with a radius a little larger than the outside edge of your kaleidoscope.

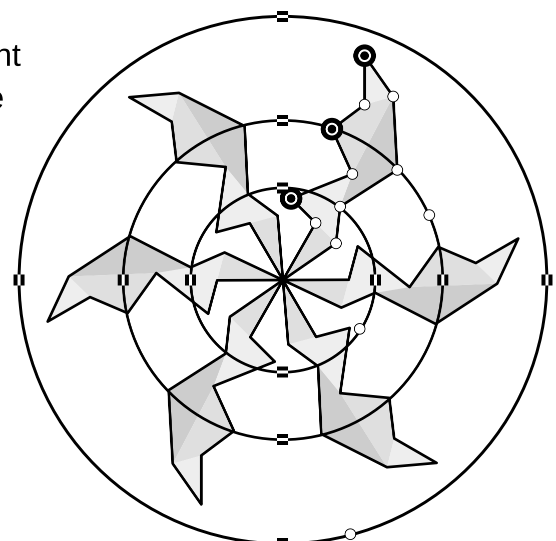
c. Using the Circle tool, construct another circle with its center at the center of your kaleidoscope, but this time let the radius be about half the radius of your kaleidoscope. Repeat for a circle with a radius about one-third the radius of your kaleidoscope.

Note: Make sure you release your mouse in a blank space between two arms of your kaleidoscope. You do not want the outside control points of your circles to be constructed on any part of your kaleidoscope.

Step 6: Animate points of your kaleidoscope on the three circles.

a. Click on the Selection Arrow tool. Click in any blank space to deselect objects.

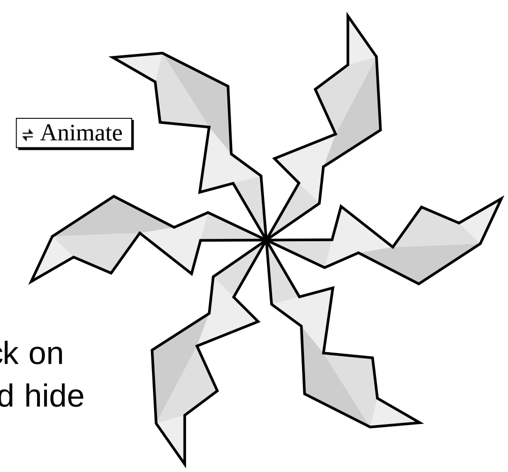
b. Hold down the Shift key. Select one point on the original polygon near the outside circle and select the outside circle (do not click on one of the control points of the circle).



c. While you continue to hold down the Shift key, select a point near the middle circle and then select the middle circle. Select a point near the smallest circle and select the smallest circle.

d. Go to the **Edit** menu, choose **Action Button**, and drag to the right and choose **Animation**. Click on **Animate** in the **Animate** dialog box.

e. When the Animate button appears, double click on it to start the animation. Watch your kaleidoscope turn!

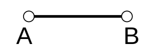


f. Click in any blank space to stop the animation. To hide all the points, click on the Point tool. Go to the **Edit** menu and choose **Select All Points**. Go to the **Display** menu and choose **Hide Points**. Click on the Circle tool, select all the circles, and hide them.

Constructing a Pantograph

A *pantograph* is a simple mechanical device that uses two pens to copy and enlarge or reduce drawings or maps. Whenever needed, use the detailed instructions marked by . Displaying the labels for points may be helpful in this construction. You can choose **Preferences** from the **Display** menu and check the **Auto Show Labels for Points** box, or you can use the Text tool to click on the points to show the labels as you go. *Make sure you did each step correctly before you go on to the next step.*

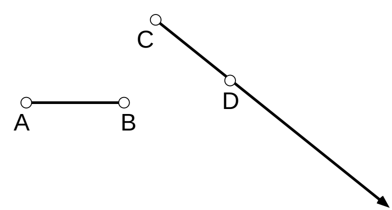
Step 1: Open a new sketch and construct $\overline{\textbf{AB}}$.



a. Go to the **File** menu and choose **New Sketch**.

b. Click on the Segment tool . Construct a segment, $\overline{\textbf{AB}}$, in your sketch.
Segment **AB** is not part of the pantograph but is a control segment that will make your pantograph adjustable.

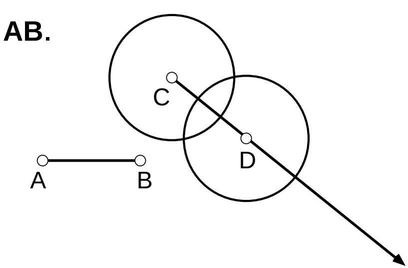
Step 2: Construct $\overrightarrow{\textbf{CD}}$.



a. Press and hold down the mouse button on the Segment tool . Drag to the right and click on the Ray tool to highlight it.

b. Construct a ray similar to the one at right.

Step 3: Construct circles with centers **C** and **D** and radius **AB**.



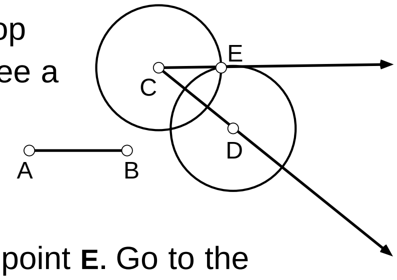
a. Click on the Selection Arrow tool . Click in any blank space to deselect objects.

b. Hold down the Shift key. Select point **C** and $\overline{\textbf{AB}}$. Go to the **Construct** menu and choose **Circle By Center+Radius**.

c. Click in any blank space to deselect objects. Hold down the Shift key. Select point **D** and $\overline{\textbf{AB}}$. Go to the **Construct** menu and choose **Circle By Center+Radius**.

Step 4: Construct $\overrightarrow{\mathbf{CE}}$, where **E** is one of the intersection points of these circles, as shown.

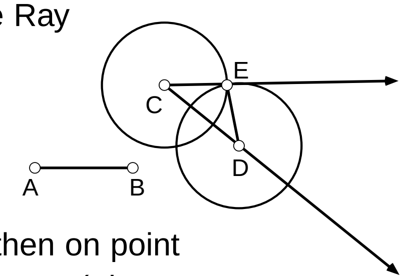
a. Using the Selection Arrow tool, click on the top intersection point of the two circles. You should see a point appear there. Use the Text tool to label it **E**, if needed.



b. Click in any blank space to deselect objects. Hold down the Shift key. Select point **C** and then point **E**. Go to the **Construct** menu and choose **Ray**.

Step 5: Construct $\overline{\mathbf{CE}}$ and $\overline{\mathbf{DE}}$.

a. Press and hold down the mouse button on the Ray tool. Drag to the right and click on the Segment tool to highlight it. Click on the Selection Arrow tool. Click in any blank space to deselect objects.

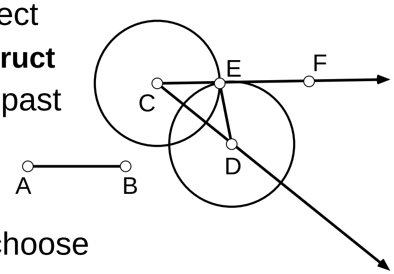


b. Hold down the Shift key. Click on point **C** and then on point **E**. Go to the **Construct** menu and choose **Segment**. (The segment you just constructed overlaps the ray.)

c. Click in any blank space to deselect objects. Hold down the Shift key. Click on point **E** and then on point **D**. Go to the **Construct** menu and choose **Segment**.

Step 6: Construct $\overline{\mathbf{EF}}$, where **F** is any point past **E** on $\overrightarrow{\mathbf{CE}}$.

a. Click in any blank space to deselect objects. Select $\overrightarrow{\mathbf{CE}}$ (click to the right of point **E**). Go to the **Construct** menu and choose **Point On Object**. Drag point **F** past point **E** if necessary.



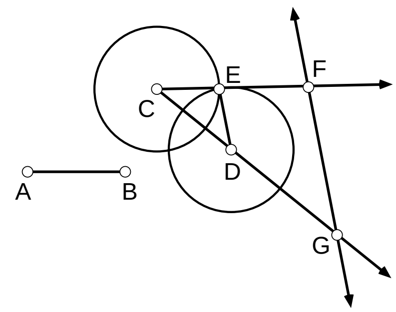
b. Hold down the Shift key. Click on point **E** and then on point **F**. Go to the **Construct** menu and choose **Segment**.

Step 7: Construct a line through point **F** parallel to $\overline{DE}$ and also construct point **G** where the new line and $\overrightarrow{CD}$ intersect.

a. Click in any blank space to deselect objects. Hold down the Shift key. Click on point **F** and then on $\overline{DE}$.

b. Go to the **Construct** menu and choose **Parallel Line**.

c. Click on the intersection of this new line and $\overrightarrow{CD}$ to construct point **G**.

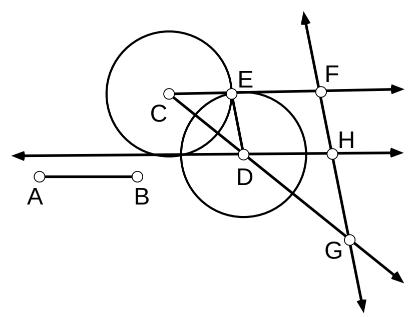


Step 8: Construct a line through point **D** parallel to $\overline{EF}$.

a. Click in any blank space to deselect objects. Hold down the Shift key. Click on point **D** and then on $\overline{EF}$.

b. Go to the **Construct** menu and choose **Parallel Line**.

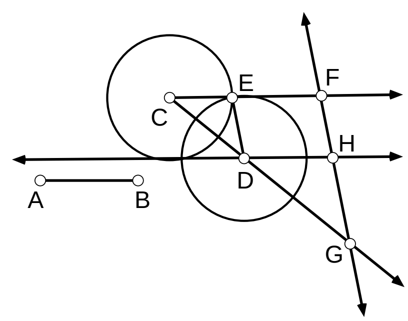
c. Click on the intersection of this parallel line and line **FG** to construct point **H**.



Step 9: Construct $\overline{DH}$, where point **H** is the intersection of the lines constructed in steps 7 and 8.

a. Click in any blank space to deselect objects. Hold down the Shift key. Click on point **D** and then click on point **H**.

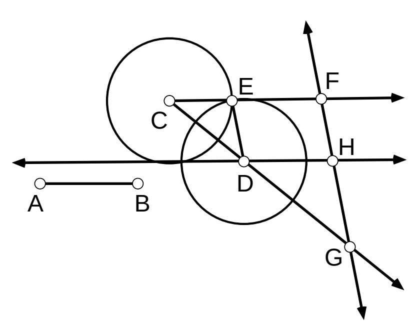
b. Go to the **Construct** menu and choose **Segment**.



Step 10: Construct $\overline{FG}$.

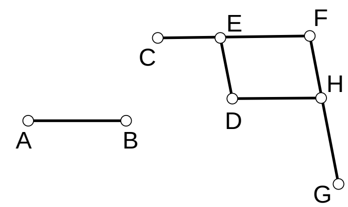
a. Click in any blank space to deselect objects. Hold down the Shift key. Click on point **F** and then on point **G**.

b. Go to the **Construct** menu and choose **Segment**.



Step 11: Hide the circles, lines, and rays so that your pantograph consists only of segments.

a. Click in any blank space to deselect objects. Hold down the Shift key. Click on the circles, lines, and rays you want to hide.



b. Go to the **Display** menu and choose **Hide Objects**.

c. Repeat step b if you missed any objects. Your sketch should look similar to the one at right.

Step 12: Trace points **D** and **G** and drag point **D** to write something.

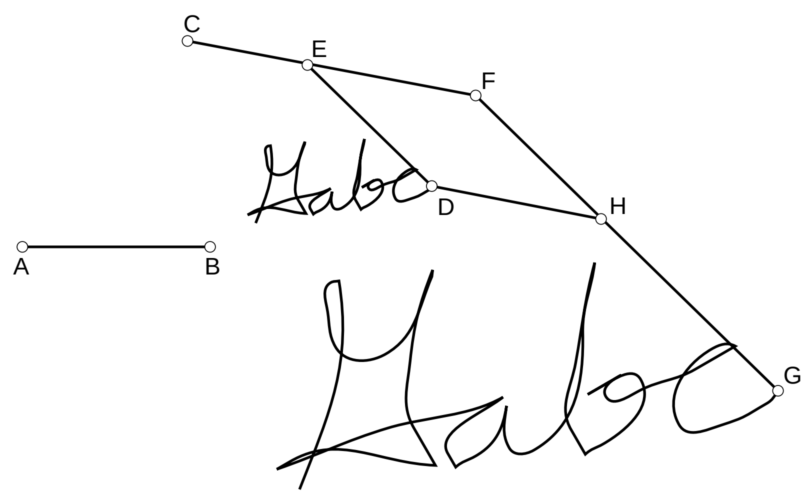
a. Click in any blank space to deselect objects. Hold down the Shift key. Select points **D** and **G**.

b. Go to the **Display** menu and choose **Trace Points**. Click in any blank space to deselect objects.

c. Select point **D** and slowly drag. Try writing your name with your pantograph.

d. Change the length of $\overline{\textbf{AB}}$ and try writing your name again. How does this affect the tracing by point **G**?

e. Change the position of point **F**. How does this affect the tracing by point **G**?



Constructing a Box with Two-Point Perspective

Perspective is a way of drawing three-dimensional objects in two dimensions—on a piece of paper or a computer screen, for example—so that they appear true-to-life. Objects that are farther away appear smaller. Perspective drawings take advantage of this principle to make flat drawings appear to have depth. Follow the directions below to draw a box with two-point perspective. Labels are shown to make the directions clearer, but you don't need labels in your drawing. Whenever needed, use the detailed instructions marked by . *Make sure you did each step correctly before you go on to the next step.*

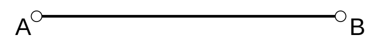
Step 1: Open a new sketch.

a. Go to the **File** menu and choose **New Sketch**.

Step 2: Draw a long horizontal segment, $\overline{\mathbf{AB}}$.

a. Click on the Segment tool .

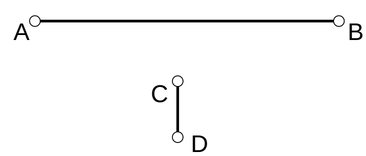
b. Press and drag to draw a segment in your sketch. (If you hold down the Shift key while you drag, it will be easier to make your segment horizontal.)



Note: This will be your horizon line, and its endpoints will be the vanishing points of your perspective box.

Step 3: Draw a short, vertical segment, $\overline{\mathbf{CD}}$, below your horizon line.

a. Press and drag with the Segment tool to draw a vertical segment below $\overline{\mathbf{AB}}$.

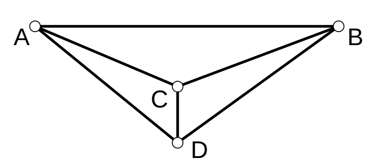


Note: This will be the front edge of your box.

Step 4: Construct $\overline{\mathbf{CA}}$, $\overline{\mathbf{DA}}$, $\overline{\mathbf{CB}}$, and $\overline{\mathbf{DB}}$.

a. Click on the Selection Arrow tool . Click in any blank space to deselect objects.

b. Hold down the Shift key. Select point **C** and then point **A**.



c. Go to the **Construct** menu and choose **Segment**.

d. Click in any blank space to deselect objects. Hold down the Shift key. Select point **D** and then point **A**.

e. Go to the **Construct** menu and choose **Segment**.

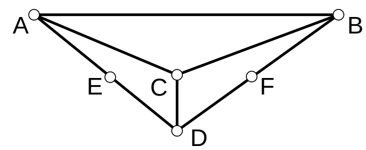
f. Click in any blank space to deselect objects. Hold down the Shift key. Select point **C** and then point **B**.

g. Go to the **Construct** menu and choose **Segment**.

h. Click in any blank space to deselect objects. Hold down the Shift key. Select point **D** and then point **B**.

i. Go to the **Construct** menu and choose **Segment**.

Step 5: Construct point **E** on $\overline{\mathbf{DA}}$ and point **F** on $\overline{\mathbf{DB}}$.

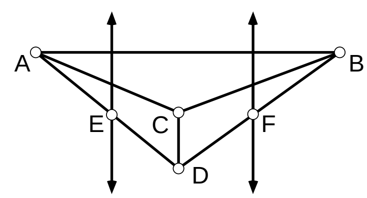


a. Click in any blank space to deselect objects. Hold down the Shift key. Select $\overline{\mathbf{DA}}$ and $\overline{\mathbf{DB}}$.

b. Go to the **Construct** menu and choose **Point On Object**.

c. Drag each point to approximately the middle of each segment.

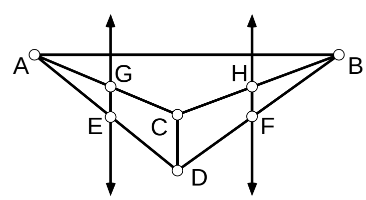
Step 6: Construct lines parallel to $\overline{\mathbf{CD}}$ through points **E** and **F**.



a. Click in any blank space to deselect objects. Hold down the Shift key. Select $\overline{\mathbf{CD}}$ and points **E** and **F**.

b. Go to the **Construct** menu and choose **Parallel Line**.

Step 7: Construct the intersection points **G** and **H**.

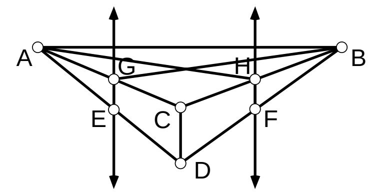


a. Click in any blank space to deselect objects. Hold down the Shift key. Select the parallel line on the left and $\overline{\mathbf{AC}}$.

b. Go to the **Construct** menu and choose **Point At Intersection**.

c. Construct point **H** on the parallel line on the right in the same way.

Step 8: Construct $\overline{GB}$ and $\overline{HA}$.

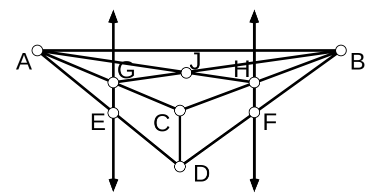


a. Click in any blank space to deselect objects. Hold down the Shift key. Select point **G** and then point **B**.

b. Go to the **Construct** menu and choose **Segment**.

c. Construct $\overline{HA}$ in the same way.

Step 9: Construct point **J** at the intersection of $\overline{GB}$ and $\overline{HA}$



a. Click in any blank space to deselect objects. Hold down the Shift key. Select $\overline{GB}$ and then $\overline{HA}$.

b. Go to the **Construct** menu and choose **Point At Intersection**.

Step 10: Construct $\overline{GJ}$, $\overline{JH}$, $\overline{HF}$, $\overline{FD}$, $\overline{DE}$, $\overline{EG}$, $\overline{GC}$, and $\overline{CH}$.

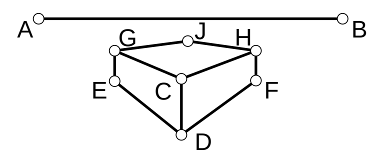
Note: These segments are the remaining edges of your box. Each overlaps the larger segment or line it is a part of.

a. Click in any blank space to deselect objects. Hold down the Shift key. Select point **G** and then point **J**.

b. Go to the **Construct** menu and choose **Segment**.

c. Construct $\overline{JH}$, $\overline{HF}$, $\overline{FD}$, $\overline{DE}$, $\overline{EG}$, $\overline{GC}$, and $\overline{CH}$ in the same way.

Step 11: Hide all lines and segments that are not part of your box.

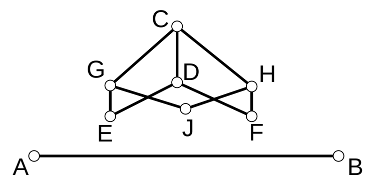


a. Click in any blank space to deselect objects. Hold down the Shift key. Click outside your box on the part of the line or segment you want to hide.

b. Go to the **Display** menu and choose **Hide Straight Objects**.

Step 12: Move various parts of your box (you can always undo) until you find the view you like best.

Extension: If you moved the front edge of your box above your horizontal segment, $\overline{\mathbf{AB}}$, you should have noticed that your box has no bottom. Use the following steps to create the missing faces and bottom.



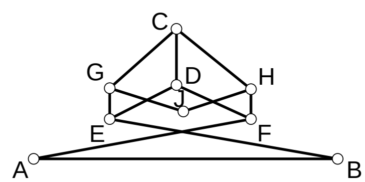
Step 13: Select $\overline{\mathbf{AB}}$ and drag it below your box.

Step 14: Construct $\overline{\mathbf{EB}}$ and $\overline{\mathbf{FA}}$.

a. Click in any blank space to deselect objects. Hold down the Shift key. Select point **E** and then point **B**.

b. Go to the **Construct** menu and choose **Segment**.

c. Construct $\overline{\mathbf{FA}}$ in the same way.



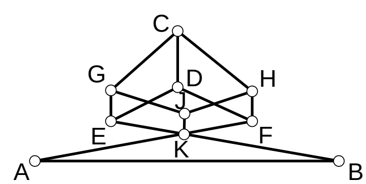
Step 15: Construct point **K** and $\overline{\mathbf{KJ}}$.

a. Click in any blank space to deselect objects. Hold down the Shift key. Select $\overline{\mathbf{EB}}$ and $\overline{\mathbf{FA}}$.

b. Go to the **Construct** menu and choose **Point At Intersection**.

c. Click in any blank space to deselect objects. Hold down the Shift key. Select points **J** and **K**.

d. Go to the **Construct** menu and choose **Segment**.

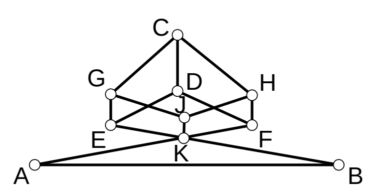


Step 16: Construct $\overline{\mathbf{EK}}$ and $\overline{\mathbf{FK}}$.

a. Click in any blank space to deselect objects. Hold down the Shift key. Select point **E** and then point **K**.

b. Go to the **Construct** menu and choose **Segment**.

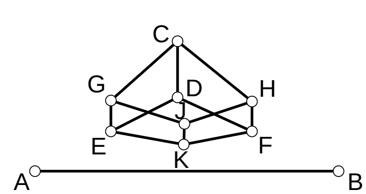
c. Construct $\overline{\mathbf{FK}}$ in the same way.



Step 17: Hide unwanted points and segments.

a. Click in any blank space to deselect objects. Hold down the Shift key. Select points and segments you want to hide.

b. Go to the **Display** menu and choose **Hide Objects**.

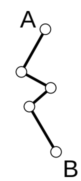


You have a *complete* box with two-point perspective!

Constructing Tessellations by Translations

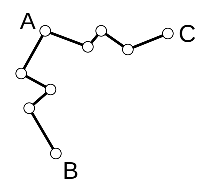
Follow the directions below to produce a tessellation by translations. You may want to label the points we have labeled in the instructions to more clearly identify the needed points for marking the vectors. Use the Text tool to click on the points you wish to label. Whenever needed, use the detailed instructions marked by . *Make sure you did each step correctly before you go on to the next step.*

Step 1: Open a new sketch. Construct two adjacent sides of a fractured parallelogram.



a. Go to the **File** menu and choose **New Sketch**.

b. Click on the Segment tool and use it to draw a vertical fractured side for your fractured parallelogram (similar to the one from **A** to **B** shown at right). Use the Text tool to label points when needed.

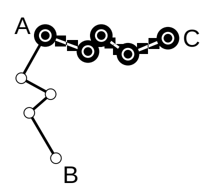


c. Click in any blank space to deselect objects.

d. Start at point **A** and draw a fractured horizontal side similar to the one shown here from **A** to **C**.

Step 2: Translate one side of the fractured parallelogram.

a. Click in any blank space to deselect objects. Click on the Selection Arrow tool . Hold down the Shift key. Select point **A** and then point **B**.



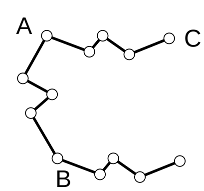
step d

b. Go to the **Transform** menu and choose **Mark Vector**.

c. Click in any blank space to deselect objects.

d. Hold down the Shift key. Select all points and segments along the fractured side from point **A** to point **C**.

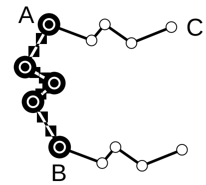
e. Go to the **Transform** menu and choose **Translate**. Choose **By Marked Vector**. Then click on **OK**. Your figure should look similar to the one shown at right.



step e

Step 3: Use a translation to complete the fractured parallelogram.

a. Click in any blank space to deselect objects. Hold down the Shift key. Select point **A** and then point **C**. Go to the **Transform** menu and choose **Mark Vector**.

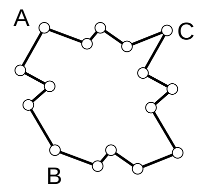


step c

b. Click in any blank space to deselect objects.

c. Hold down the Shift key. Select all points and segments along the fractured side from point **A** to point **B**.

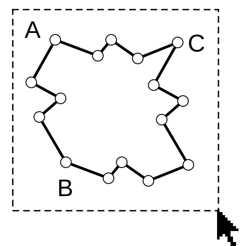
d. Go to the **Transform** menu and choose **Translate**. Choose **By Marked Vector**. Then click on **OK**. Your figure should look similar to the one shown at right.



step d

Step 4: Translate your fractured parallelogram to tessellate the plane.

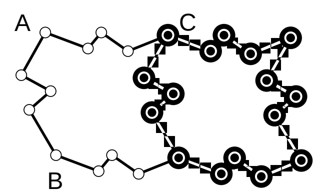
a. Use a selection marquee to select your figure.



step a

b. Go to the **Transform** menu and choose **Translate**. Choose **By Marked Vector**. Then click on **OK**. Your figure should look similar to the one shown.

c. While the new figure is selected, repeat step b so that you have three figures in a row.



step b

d. Click in any blank space to deselect objects.

e. Hold down the Shift key. Select point **A** and then point **B**.

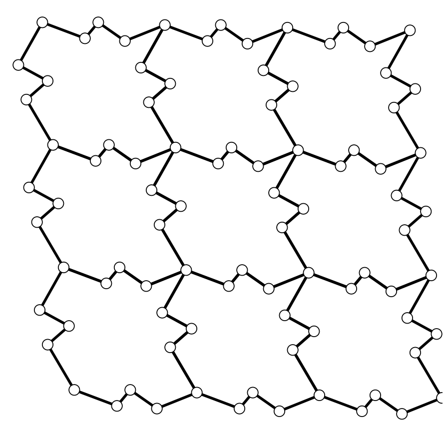
f. Go to the **Transform** menu and choose **Mark Vector**.

g. Use a selection marquee to select your entire figure.

h. Go to the **Transform** menu and choose **Translate**. Choose **By Marked Vector**. Then click on **OK**.

i. While the new row is selected, repeat step h to produce a figure similar to the one shown at right.

j. Drag a point in one of your fractured parallelograms to see what happens. Continue to drag until you have a shape you like.


step i

k. You may want to construct polygon interiors and shade them to give some contrast to your tessellation. (You can construct just one polygon interior and translate it again and again to fill all the polygons in the tessellation.)

Remember: To construct a polygon interior, select the vertices in clockwise or counterclockwise order, go to the **Construct** menu, and choose **Polygon Interior**. To change the shade or color of a polygon interior, click on it to select it, go to the **Display** menu, and choose **Shade** or **Color**. Pick a shade or color you like.

Constructing a Binary Tree Fractal

You will use Sketchpad to record a recursive script to create the first stages of the binary tree fractal. Follow the directions below. Labels are shown to make the directions clearer, but you don't need labels in your drawing. Whenever needed, use the detailed instructions marked by . *Make sure you did each step correctly before you go on to the next step.*

Step 1: Open a new sketch and a new script and start recording the script.

a. Go to the **File** menu and choose **New Sketch**.

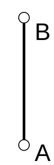
b. Go to the **File** menu and choose **New Script**.

c. Click on **REC** in the **Script** window.

Step 2: Draw a vertical segment (approximately 2 inches long) in your sketch.

a. Click on the Segment tool .

b. Draw a vertical segment in your sketch. (Start near the bottom of your sketch and drag upward to point **B**.)



Step 3: Mark point **A** as a center.

a. Click on the Selection Arrow tool .

b. Select point **A**. Go to the **Transform** menu and choose **Mark Center**.

Step 4: Dilate point **B** upward by a scale factor of 3 to 2.

a. Select point **B**.

b. Go to the **Transform** menu and choose **Dilate**.

c. In the **Dilate** dialog box, enter 3 as New and 2 as Old. Click on **OK**.

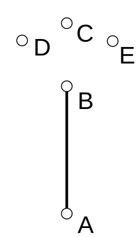
Dilate Dialog Box (Macintosh)

Step 5: Mark point **B** as a center. Rotate point **C** by an angle of 45°. Then rotate point **C** by an angle of –45°.

a. Select point **B**. Go to the **Transform** menu and choose **Mark Center**.

b. Select point **C**. Go to the **Transform** menu and choose **Rotate**. In the **Rotate** dialog box, enter 45° as the angle of rotation.

c. Repeat step b, except this time enter –45° as the angle of rotation.



Now you will set up the recursive steps to create new stages for your fractal.

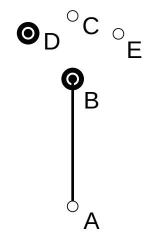
Step 6: Loop your script on points **B** and **D**.

a. Click in any blank space to deselect objects.

b. Hold down the Shift key. Select point **B** and then point **D**.

c. Click back on your **Script** window box and click on **Loop**.

d. Click back on your **Sketch** window box and click in any blank space to deselect objects.

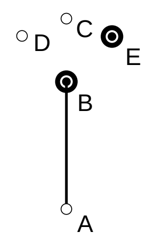


Step 7: Loop your script on points **B** and **E**.

a. Hold down the Shift key. Select point **B** and then point **E**.

b. Click back on your **Script** window and click on **Loop**.

c. Click back on your **Sketch** window and click in any blank space to deselect objects.



Step 8: Hide all points.

a. Click on the Point tool.

b. Go to the **Edit** menu and choose **Select All Points**.

c. Go to the **Display** menu and choose **Hide Points**.

Now you can view one of several stages of your binary tree fractal.

Step 9: Stop recording your script.

a. Click on the **Script** window.

b. Click on **Stop**.

Step 10: Play your script.

a. Go to the **File** menu and choose **New Sketch**.

b. Click on the Point tool. Hold down the Shift key. Construct two points in your sketch, constructing first the bottom point and then the top point.

c. Click back on your **Script** window and click on **Fast**.

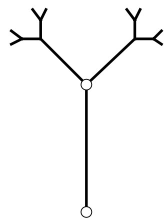
d. In the **Recursion** dialog box, enter 3 and click on **OK** or **Play**. You can experiment with the depth of recursion later.

Recursion Dialog Box (Windows)

You should have a nice binary tree fractal in your sketch. Play your script again and experiment with the depth of recursion.

Follow-up Constructions

Try writing your own script to produce a fractal of a different design!



Constructing a Sierpinski Gasket Fractal

You will use Sketchpad to record a recursive script to create the first stages of the Sierpinski Gasket fractal. Follow the directions below. Labels are shown to make the directions clearer, but you don't need labels in your drawing. Whenever needed, use the detailed instructions marked by . In this construction, it's important to follow the selection order exactly as it's described. If your script doesn't work correctly, your best bet is to start over in a new sketch with a new script, being very careful about selection order. *Make sure you did each step correctly before you go on to the next step.*

Step 1: Open a new sketch and a new script and start recording the script.

a. Go to the **File** menu and choose **New Sketch**.

b. Go to the **File** menu and choose **New Script**.

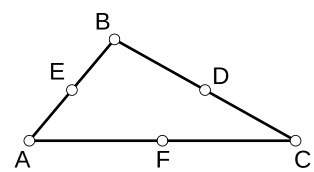
c. Click on **REC** in the **Script** window.

Step 2: Construct a triangle, Δ**ABC**, and construct the midpoints of its sides.

a. Use the Segment tool to construct a triangle.

b. Hold down the Shift key. Click on the Selection Arrow tool . Click on the three sides of your triangle to select them.

c. Go to the **Construct** menu and choose **Point At Midpoint**.

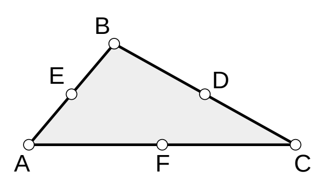


Step 3: Construct the polygon interior of triangle **ABC**. Shade it a light shade.

a. Click in any blank space to deselect objects.

b. Hold down the Shift key. Select the vertices of triangle **ABC** *in that order.* Go to the **Construct** menu and choose **Polygon Interior**.

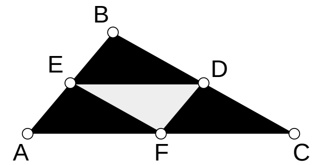
c. Select the polygon interior. Go to the **Display** menu and choose **Shade**. Pick a light shade.



Step 4: Construct the polygon interiors of triangles **AEF**, **EBD**, and **FDC**.

a. Click in any blank space to deselect objects.

b. Hold down the Shift key. Select the points **A**, **E**, and **F** *in that order.* Go to the **Construct** menu and choose **Polygon Interior**.



c. Select the polygon interior. Go to the **Display** menu and choose **Shade**. Pick a dark shade.

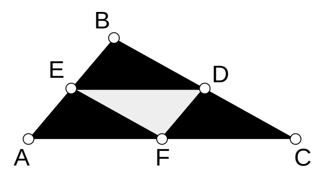
d. Construct the interiors of △**EBD** and △**FDC**, *in that order,* in the same way.

Now you will set the recursive steps to create new stages for your fractal.

Step 5: Select points **A**, **E**, and **F** and click on **Loop**. Repeat for points **E**, **B**, and **D** and points **F**, **D**, and **C**.

a. Click in any blank space to deselect objects.

b. Hold down the Shift key. Select points **A**, **E**, and **F**, *in that order.*



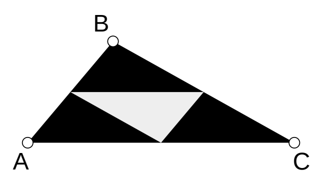
c. Click on your **Script** window and click on **Loop**.

d. Loop on points **E**, **B**, and **D** and points **F**, **D**, and **C** in the same way. Be careful to follow this selection order.

Step 6: Hide the interior of the large triangle **ABC**. Then hide all midpoints and segments.

a. Click in any blank space to deselect objects.

b. Click in the lightly shaded area in the center of △**ABC**. Go to the **Display** menu and choose **Hide Polygon**.



c. Hold down the Shift key. Select the sides of △**ABC** and all the points and midpoints of the triangles. Go to the **Display** menu and choose **Hide Objects**.

Step 7: Stop your script.

a. Click on the **Script** window and click on **Stop**.

Now you can view your Sierpinski Gasket fractal.

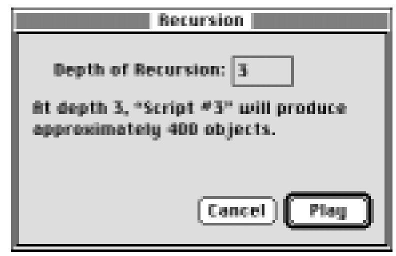
Step 8: Play your script.

a. Open a new sketch.

b. Hold down the Shift key and use the Point tool to construct three points (vertices of a triangle) in your sketch. These are the requirements (givens) of your script. They should be selected.

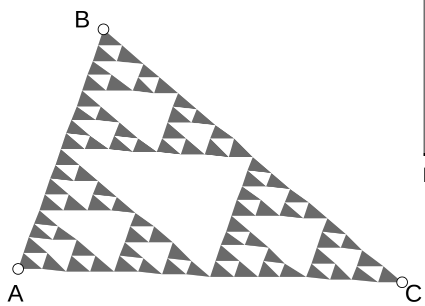
c. Click back on your **Script** window and click on **Fast.**

d. In the **Recursion** dialog box, enter 3 and click on **OK** or **Play.** You can experiment with the depth of recursion later.



Recursion Dialog Box (Macintosh)

You should have a nice Sierpinski Gasket fractal sketch. Play your script again and experiment with the depth of recursion.



Follow-up Constructions

Try writing your own script to produce a fractal of a different design.

Constructing a Dragon Fractal

If you are familiar with the book *Jurassic Park,* you may have noticed that a new stage of the dragon fractal is found at the beginning of each chapter. You will use Sketchpad to record a recursive script to create the first stages of the dragon fractal. Follow the directions below. Labels are shown to make the directions clearer, but you don't need labels in your drawing. Whenever needed, use the detailed instructions marked by . *Make sure you did each step correctly before you go on to the next step.*

Step 1: Open a new sketch and a new script and start recording the script.

a. Go to the **File** menu and choose **New Sketch**.

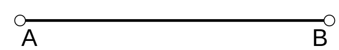
b. Go to the **File** menu and choose **New Script**.

c. Click on **REC** in the **Script** window.

Step 2: Draw a horizontal segment (approximately 2 inches long) in your sketch.

a. Click on the Segment tool .

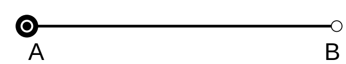
b. Press and drag to draw a horizontal segment in your sketch. (Holding down the Shift key while you drag makes it easier to make your segment horizontal.)



Step 3: Mark point **A** as a center.

a. Click on the Selection Arrow tool .

b. Select point **A**. Go to the **Transform** menu and choose **Mark Center**.

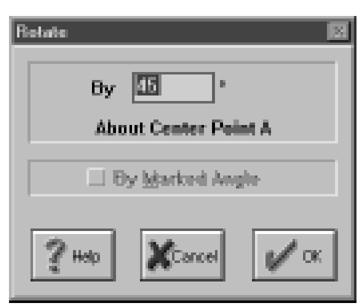


Step 4: Rotate $\overline{\textbf{AB}}$ by an angle of 45° about point **A**.

a. Select $\overline{\textbf{AB}}$.

b. Go to the **Transform** menu and choose **Rotate**.

c. In the **Rotate** dialog box, choose **By Fixed Angle** and enter 45. Click on **OK**.



Rotate Dialog Box (Windows)

Step 5: Mark point **B** as a center and rotate $\overline{AB}$ by an angle of –45° about point **B**.

a. Select point **B**. Go to the **Transform** menu and choose **Mark Center**.

b. Select $\overline{AB}$.

c. Go to the **Transform** menu and choose **Rotate**.

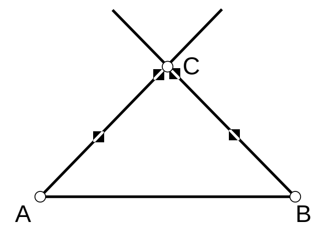
d. In the **Rotate** dialog box, choose **By Fixed Angle** and enter –45. Click on **OK**.

Rotate Dialog Box (Macintosh)

Step 6: Construct the intersection point of the rotated segments.

a. Hold down the Shift key. Select the two new segments.

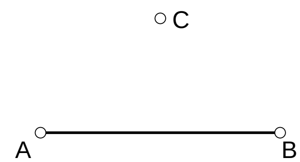
b. Go to the **Construct** menu and choose **Point At Intersection**.



Step 7: Hide the two rotated segments.

a. Hold down the Shift key. Select the two new segments.

b. Go to the **Display** menu and choose **Hide Segments**.



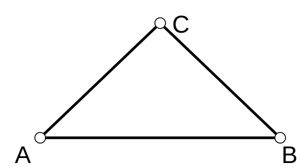
Step 8: Construct segments **AC** and **BC**.

a. Hold down the Shift key. Select point **A** and then point **C**.

b. Go to the **Construct** menu and choose **Segment**. Click in any blank space to deselect objects.

c. Hold down the Shift key. Select point **B** and then point **C**.

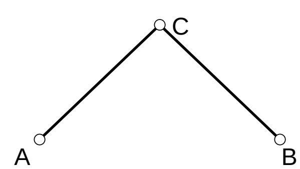
d. Go to the **Construct** menu and choose **Segment**.



Step 9: Hide segment **AB**.

a. Select $\overline{AB}$.

b. Go to the **Display** menu and choose **Hide Segment**.



Now you will set up the recursive steps to create the new stages of your fractal.

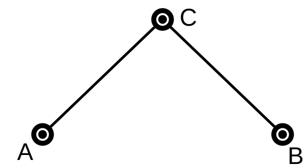
Step 10: Loop your script on points **A** and **C** and then loop on points **B** and **C**.

a. Hold down the Shift key. Select point **A** and then point **C**.

b. Click back on your **Script** window and click on **Loop**.

c. Click back on your **Sketch** window and click in any blank space to deselect objects.

d. Loop on points **B** and **C** in the same way.



Step 11: Hide all points.

a. Click on the Point tool ⊡.

b. Go to the **Edit** menu and choose **Select All Points**.

c. Go to the **Display** menu and choose **Hide Points**.

Step 12: Stop recording your script.

a. Click on the **Script** window.

b. Click on **Stop**.

Step 13: Play your script.

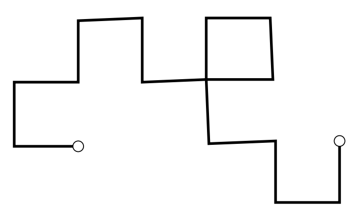
a. Go to the **File** menu and choose **New Sketch**.

b. Click on the Point tool ⊡. Hold down the Shift key. Construct two points in your sketch.

c. Click back on your **Script** window and click on **Fast**.

d. In the **Recursion** dialog box, enter 3 and click on **OK** or **Play**. You can experiment with the depth of recursion later.

Recursion Dialog Box (Windows)



You should have the beginning stages of a dragon fractal in your sketch. Play your script again and experiment with the depth of recursion.

Follow-up Constructions

Try writing your own script to produce a fractal of a different design!

Shortcuts with The Geometer's Sketchpad

There are two types of shortcuts you can perform with Sketchpad. The first type uses the Command key (⌘) on Macintosh keyboards or the Control key (Ctrl) on Windows keyboards and another keystroke. Each shortcut is listed with the appropriate menu item.

Keystroke	Result	Keystroke	Result
⌘ Z or Ctrl Z	Undo	⌘ K or Ctrl K	Show or hide labels
⌘ R or Ctrl R	Redo	⌘ T or Ctrl T	Trace objects
⌘ A or Ctrl /	Select all	⌘ I or Ctrl I	Construct point at intersection
⌘ N or Ctrl N	Open a new sketch	⌘ M or Ctrl M	Construct point at midpoint
⌘ O or Ctrl O	Open dialog box for files	⌘ L or Ctrl L	Construct segment, ray, or line
⌘ W or Ctrl W	Close file	⌘ P or Ctrl P	Construct interior of polygon, circle, or arc sector
⌘ Q or Ctrl Q	Quit or exit Sketchpad	⌘ F or Ctrl F	Mark a center
⌘ S or Ctrl S	Save file	⌘ G or Ctrl G	Mark a mirror
⌘ H or Ctrl H	Hide objects		

The second type of shortcut uses the Sketchpad tools. When performed correctly, such shortcuts result in constructions that are just as sound as those made by using the commands from the menu.

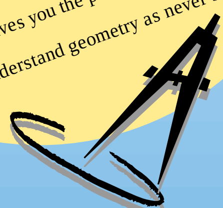
Objective	Technique
Mark a mirror	Using the Selection Arrow tool, double click on the segment, ray, or line you wish to be the mirror.
Mark a center	Using the Selection Arrow tool, double click on the point you wish to be the center.
Construct a segment, ray, or line through two given points	Using the Segment, Ray, or Line tool, press on the first point and then drag, releasing on the second point.
Construct a circle using two given points, one as the center and the other as a point on the circle.	Using the Circle tool, press on the point you wish to be the center and then drag, releasing on the second point.
Construct the intersection point(s) of two given objects	Using the Selection Arrow tool, or using the Point tool, click on the point of intersection. (This method is not as foolproof for beginners.) Note: you can also draw segments, rays, lines, and circles to intersections. You don't have to construct a point first.

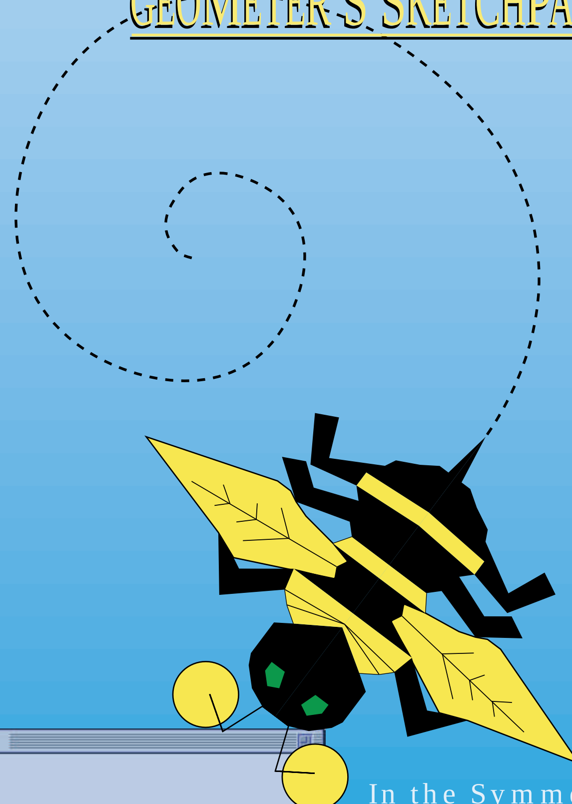
Geometry Activities for Middle School Students

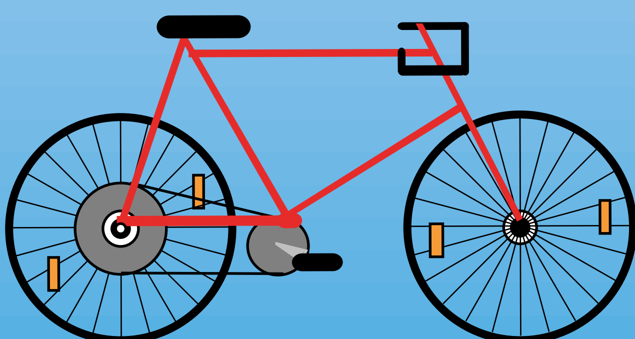
with THE GEOMETER'S SKETCHPAD®

Dynamic Geometry® for the 21st Century

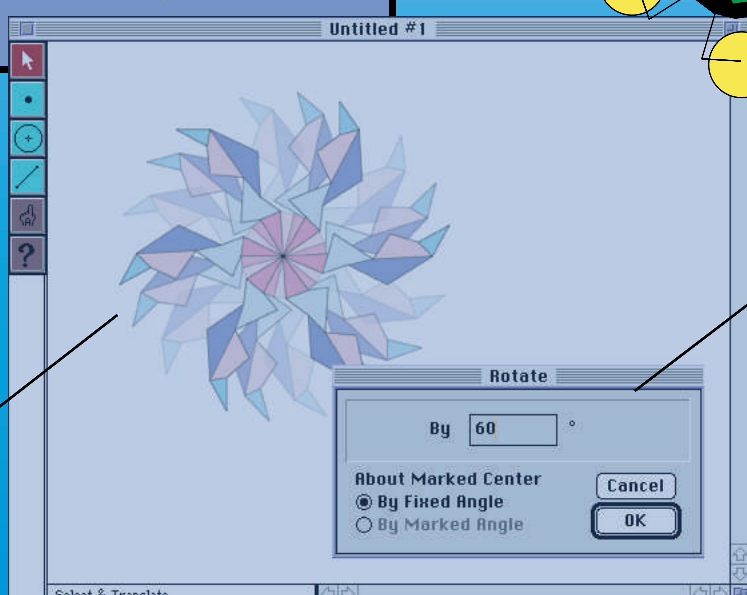
The Geometer's Sketchpad® is a dynamic geometry construction and exploration tool for students, teachers, researchers, artists, and other geometry lovers. With Sketchpad™ you can design figures with freehand drawing and precise construction techniques. Then you can transform, manipulate, and distort your figure interactively, while preserving all of its geometric relationships. Dynamic interaction gives you the power to explore, analyze, and understand geometry as never before.







Harness the power of Sketchpad to bring geometry concepts to life in your middle school classroom.



The CD-ROM that accompanies this book contains many examples of dazzling student projects as well as all the sketches required for the activities and teacher demonstrations.

In the Symmetry unit activities, students learn how to test figures for symmetry and how to construct symmetric figures using transformations. In other units, students explore topics ranging from simple line and angle relationships to such complex constructions—made easy with Sketchpad—as the dragon-curve fractal.

KEY CURRICULUM PRESS
Innovators in Mathematics Education

ISBN 1-55953-299-8
90000>
9 781559 532990